Net torque on star

- **Field drives strong inflow inside R_m.**
- **Accretion torque equals angular momentum flux through R_m:**

$$T_{acc} = \dot{M} R_m^2 \Omega_K(R_m)$$

- **Hence net torque on star is:**

$$T_{mag} + T_{acc} = \frac{4\pi B_0^2 R_*^6}{3\mu_0}\left(R_m^{-3} - 2R_c^{-3/2} R_m^{-3/2}\right) + \dot{M} R_m^2 \Omega_* \left(\frac{R_m}{R_c}\right)^{-3/2}$$

Net torque must be zero in spin equilibrium

Spin-equilibrium conditions

- **Rearrange expression for zero net torque :**

$$\left(\frac{R_m}{R_c}\right)^{-7/2} - 2\left(\frac{R_m}{R_c}\right)^{-2} = -\frac{3}{4}\frac{\mu_0 \dot{M} (G M_*)^{1/2}}{\pi B_0^2 R_*^6} R_c^{7/2}$$

- **...to match form used to define R_m:**

$$\left(\frac{R_m}{R_c}\right)^{-7/2} - \left(\frac{R_m}{R_c}\right)^{-2} = \frac{1}{8}\frac{\mu_0 \dot{M} (G M_*)^{1/2}}{\pi B_0^2 R_*^6} R_c^{7/2}$$

- **Subtract 1st equation from twice 2nd equation:**

$$R_m^{-7/2} = \frac{\mu_0 \dot{M} (G M_*)^{1/2}}{\pi B_0^2 R_*^6}$$

Spin-equilibrium values of R_m and R_c

- **Subtract and substitute R_m:**

$$R_m^{-2} = \frac{7}{8}\frac{\mu_0 \dot{M} (G M_*)^{1/2}}{\pi B_0^2 R_*^6} R_c^{3/2} = \frac{7}{8} R_m^{-7/2} R_c^{3/2}$$

$$\Rightarrow R_c^{3/2} = \frac{8}{7} R_m^{3/2}, \text{ i.e. } R_c = \left(\frac{8}{7}\right)^{2/3} R_m$$

$$\text{i.e. } \boxed{R_c = \left(\frac{8}{7}\right)^{2/3} R_m \approx 1.093\, R_m}$$

$$\text{where } \boxed{R_m = \left[\frac{\pi B_0^2 R_*^6}{\mu_0 \dot{M} (G M_*)^{1/2}}\right]^{2/7}}$$

Stability of spin equilibrium

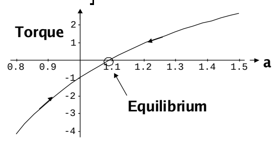
- **If star spins slower than equilibrium, then:**

$$\frac{R_c}{R_m} \equiv a > \left(\frac{8}{7}\right)^{2/3}$$

- **Substitute into expression for net torque on star:**

$$3(T_{mag} + T_{acc})\left[\frac{\mu_0}{\pi B_0^2 R_*^6 \dot{M}^6 (G M_*)^3}\right]^{1/7} = 7 - 8a^{-3/2}$$

Torque is positive when star is spinning slower than equilibrium, and vice versa => system is driven toward equilibrium.



Typical spin-equilibrium values

$$R_c = 1.076 \times 10^{10} (b_0)^{4/7} (r_*)^{12/7} (m_*)^{-1/7} (\dot{m})^{-2/7}\ \text{m}$$

$$\frac{R_c}{R_*} = 6.19 (b_0)^{4/7} (r_*)^{5/7} (m_*)^{-1/7} (\dot{m})^{-2/7}$$

$$P_{rot} = 7.031 (b_0)^{6/7} (r_*)^{18/7} (m_*)^{-5/7} (\dot{m})^{-3/7}\ \text{day}$$

$$\text{where } b_0 \equiv \left(\frac{B_0}{0.1\,T}\right), r_* \equiv \left(\frac{R_*}{2.5\, R_{Sun}}\right),$$

← $1\ M_{Sun}$ TTS at age = 1 Myr

$$m_* \equiv \left(\frac{M_*}{M_{Sun}}\right), \dot{m} \equiv \left(\frac{\dot{M}}{10^{-8} M_{Sun}\,\text{y}^{-1}}\right)$$

The boundary layer

- **In non-magnetic systems, accretion occurs directly on to central object.**
- **Just above surface, material orbits at Keplerian rate.**
- **Half the available gravitational potential energy is radiated by the disc.**
- **What about the other half?**
  - Rotational energy of star
  - Internal energy of star
  - Re-radiated in boundary layer

Energetics of the impact region

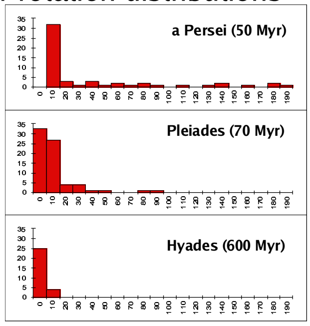
- **Material from disrupted inner disc will be channelled along field lines to impact zones on stellar surface.**
- **Less near-IR emission (2 - 10 μm) from inner disc.**
- **More energy to dissipate than in non-magnetic case:**
  - Work done by magnetic torque -- dissipated in inner disc?
  - Re-radiation from stellar impact zones (hot spots).
  - Increase in stellar internal energy.
- **Stellar magnetic field geometry controls:**
  - Location of impact zones
  - Braking efficiency
  - Nature and direction of outflows

Open cluster rotation distributions

Colour range:
0.55 < B-V < 0.85.

Mass range:
$1.0 > M / M_{sun} > 0.8$

Data taken from
Soderblom et al (1993)



v sin i (km/sec)

Cluster rotation distributions

- **α Persei (50 Myr), Pleiades (70 Myr):**
  - Narrow low-Ω peak at P_{rot} ~ 7-10 days
  - Extended high-Ω tail of ultra-fast rotators (UFRs)
  - No UFRs among Pleiades G stars
- **Hyades (600 Myr):**
  - no high-Ω tail among G or K stars
  - some residual rapid rotation among M dwarfs
- **Field stars (up to 10 Gyr):**
  - Wide range of rotation rates
  - P_{rot} increases with age.
- **Present "best-buy" explanation:**
  - Pre-main sequence evolution determines initial distribution.
  - Subsequent spindown via magnetically-channelled wind.

Protostellar spin evolution

Dynamo field: $B_z = \frac{B_{0*}}{2}\left(\frac{R}{\varpi}\right)^3, \quad B_{0*} = B_{0,\mathrm{Sun}} \frac{t_{\mathrm{conv},*}\Omega_*}{t_{\mathrm{conv,Sun}}\Omega_{\mathrm{Sun}}}$

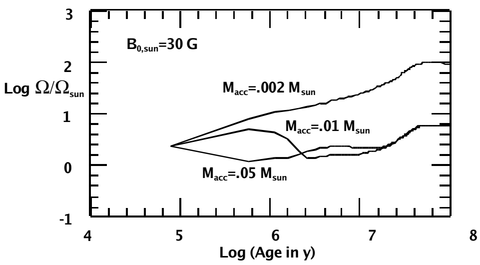
Magnetic torque: $T_m = -\frac{4\pi}{\mu_0}\int_{\varpi_m}^{\varpi_D} \varpi^2 B_\phi^+ B_z \, d\varpi$

Accretion torque: $T_a = \dot{M}\varpi_m^2 \Omega_K(\varpi_m)$

Equation of motion: $T = \frac{d}{dt}(I\Omega_*) \quad \Rightarrow \quad \dot{\Omega}_* = \frac{T - I\dot{\Omega}_*}{k^2 MR^2}$

- **Solve equation of motion numerically**
- **Use a stellar evolution code (Eggleton 1992) to compute the moment of inertia and convective turnover time at each timestep.**

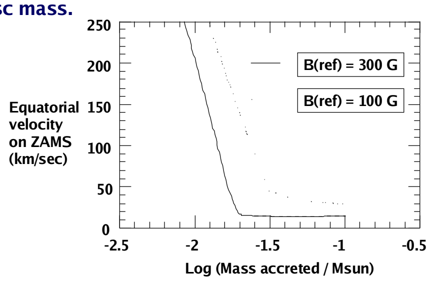
Stellar spin evolution: results



Cameron, Campbell & Quaintrell, 1995 A&A, 298, 133

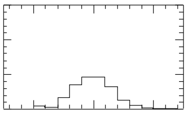
Disc mass and ZAMS rotation rate

- **Models which accrete more than 0.01 M_{sun} arrive on the ZAMS with a low rotation rate that is almost independent of disc mass.**

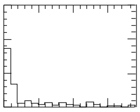


The ZAMS rotation distribution

The distribution of disc masses inferred for CTTS from 1.3 mm continuum fluxes can be approximated by a log-normal distribution:



The resulting distribution of projected equatorial rotation speeds resembles the peak-and-tail distribution found in the α Per (50 Myr) and Pleiades (70 Myr) clusters.



Spin-down on the main sequence

- **Stars continue to spin down throughout MS evolution:**
  - Rapid spindown of fast rotators in young clusters
  - Asymptotic phase (Skumanich 1972): $\Omega \sim t^{-1/2}$.
- **Angular momentum loss via hot winds:**
  - low mass-loss rates
  - thermally driven at low Ω
  - centrifugally driven at high Ω
  - magnetically channelled
  - high specific angular momentum