The spin-up timescale

- **Stellar moment of inertia:** $I = k^2 M_* R_*^2$

Squared radius of gyration ~ 0.2 for fully convective protostar

- **Accretion torque**

$$C = \frac{d}{dt}(I\Omega) = I\frac{d\Omega}{dt} + \Omega\frac{dI}{dt} = \dot{M}(GM_* R_*)^{1/2}$$

Ignore if spin-up time less than contraction time.

- **Spin-up timescale**

$$t_s \approx \frac{I\Omega_K(R_*)}{\dot{M}(GMR_*)^{1/2}} = \frac{k^2 MR_*^2}{\dot{M}(GMR_*)^{1/2}} \cdot \left(\frac{GM}{R_*^3}\right)^{1/2}$$

$$= k^2 \frac{M}{\dot{M}}$$

Typically 10^{-5} to 10^{-7} $M_{Sun}\ y^{-1}$ for classical T Tauri stars (CTTS) from L_{disc}.

$$t_s \approx 2\times10^4 - 2\times10^6\ yr$$

The contraction timescale

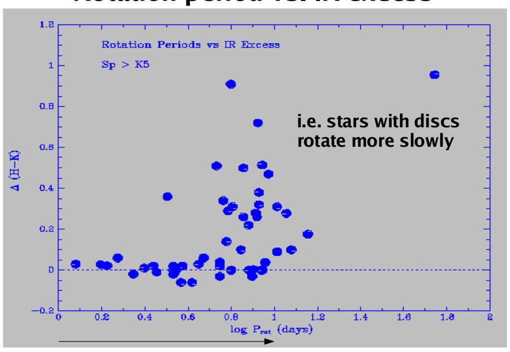
- **Gravitational contraction timescale is roughly:**

$$t_G \approx \frac{GM^2}{R_*} \cdot \frac{1}{4\pi R_*^2 \sigma T^4}$$

(about 1 to 2 Myr for a 1 M_{Sun} protostar with R = 4 R_{Sun} and T = 4500 K.)

- **i.e. $t_s \leq t_G \sim t_{visc}$.**
- **Plenty of time to spin up as disc material accretes on to star and star contracts.**
- **So why do real CTTS spin ten times more slowly than breakup???**

Rotation period vs. IR excess



1day 10days

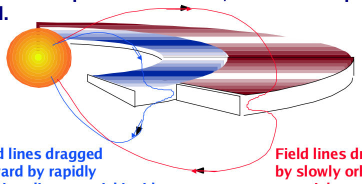
Discs and rotation

- **Bouvier 1993, Attridge & Herbst (1992) find that T Tauri stars with IR and sub-mm emission from discs rotate significantly more slowly than those without discs.**
- **Ditto Edwards et al 1993, AJ 106, 372**
- **Does the presence of a disc alter a star's early rotational evolution?**
- **Königl (1991, ApJ) suggested that magnetic drag on disc material might regulate the stellar spin rate.**

Disc brakes?

Cameron & Campbell (1993,1994) showed that a TTS can evolve into magnetic torque balance with its disc, within its Hayashi-track lifetime.

The equilibrium spin rate is about 1/10 the breakup rate, as observed.



Field lines dragged forward by rapidly orbiting disc material inside corotation radius.

Field lines dragged back by slowly orbiting disc material outside corotation radius

Disc-magnetosphere interaction

- **Dynamo generated field anchored in photosphere.**
- **Magnetosphere corotates with star.**
- **Disc cuts into magnetosphere.**
- **Field lines penetrate disc vertically:**
  - For dipole field:

$$B_z = B_0\left(\frac{R}{R_*}\right)^{-3}$$

- **Azimuthal field component:**
  - Growth due to vertical shear in u_ϕ
  - Limited by reconnection of twisted field lines in magnetosphere?
  - Simple prescription:

$$B_\phi = B_z \frac{(\Omega_K(R) - \Omega_*)}{\Omega_K(R)}$$

Vertical average

Azimuthal Lorentz force

- **Local Lorentz force**

$$\frac{1}{\mu_0}(\nabla\times \boldsymbol{B})\times \boldsymbol{B} = \frac{1}{\mu_0}(\boldsymbol{B}\cdot\nabla)\boldsymbol{B} - \nabla\left(\frac{B^2}{2\mu_0}\right)$$

Current j

zero if field is axisymmetric

- **Azimuthal tension component:**

$$\frac{1}{\mu_0}[(\boldsymbol{B}\cdot\nabla)\boldsymbol{B}]_\phi = \frac{1}{\mu_0}\left[\frac{B_R}{R}\frac{\partial}{\partial R}RB_\phi + \frac{B_\phi}{R}\frac{\partial B_\phi}{\partial\phi} + B_z\frac{\partial B_\phi}{\partial z}\right]$$

$B_R=0$

Axisymmetric

- **Integrate to define force per unit area:**

$$\frac{2}{\mu_0}B_z\int_{z=0}^{\infty}\frac{\partial B_\phi(z)}{\partial z}dz \equiv \frac{2B_zB_\phi}{\mu_0}$$

Disc has two sides!

Vertical average

Magnetic torque on disc material

- **Annulus of width ΔR feels torque:**

$$G_{\mathrm{m}} = \frac{2B_zB_\phi}{\mu_0}\,.\,2\pi R\Delta R\,.\,R \Rightarrow \frac{\partial G_{\mathrm{m}}}{\partial R} = 4\pi R^2\frac{B_z^2}{\mu_0}\frac{\Omega-\Omega_*}{\Omega}$$

Azimuthal force/area — Area of annulus — Length of moment arm

- **Diffusion equation for surface density becomes:**

$$\frac{\partial\Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}(\nu\Sigma R^{1/2})\right] + \frac{1}{R}\frac{\partial}{\partial R}\left[\frac{\Omega-\Omega_*}{\Omega}\,\frac{4B_z^2R^{5/2}}{\mu_0(GM_*)^{1/2}}\right]$$

Magnetic torque on star

- **Disc disrupted at magnetospheric radius R_m**
- **Integrate magnetic torque from R_m to infinity:**

$$T_{\mathrm{mag}} = \frac{4\pi B_0^2}{\mu_0}R_*^6\int_{R_m}^{\infty}R^{-4}\left(1-\left(\frac{R_c}{R}\right)^{-3/2}\right)dR$$

$$= \frac{4\pi B_0^2}{3\mu_0}R_*^6\left(R_m^{-3}-2R_c^{-3/2}R_m^{-3/2}\right)$$

Co-rotation radius → $R_c \equiv \left(\frac{GM_*}{\Omega_*^2}\right)^{1/3}$

- **Get magnetic spin-down torque on star if:**

$$T_{\mathrm{mag}}<0, \quad\Rightarrow \left(\frac{R_m}{R_c}\right) > 2^{-2/3}\approx 0.63$$

The disruption radius R_m

- **Differential accretion torque across annulus of width dR at radius R is:**

$$\frac{\partial G_{\mathrm{acc}}}{\partial R} = \dot M\frac{\partial}{\partial R}\left(R^2\Omega_K(R)\right) = \frac{\dot M}{2}\left(\frac{GM_*}{R}\right)^{1/2}$$

- **Disc disrupted at R_m where magnetic stresses exceed viscous stresses:**

$$\frac{\partial G_{\mathrm{acc}}}{\partial R} = \frac{\partial G_{\mathrm{m}}}{\partial R}\Rightarrow \frac{4\pi B_z^2}{\mu_0}R_m^2\frac{\Omega_K(R_m)-\Omega_*}{\Omega_K(R_m)} = \frac{\dot M}{2}\left(\frac{GM_*}{R_m}\right)^{1/2}$$

$$\Rightarrow \left(\frac{R_m}{R_c}\right)^{-7/2} - \left(\frac{R_m}{R_c}\right)^{-2} = \frac{\mu_0\dot M(GM_*)^{1/2}}{8\pi B_0^2R_*^6}R_c^{7/2}$$