The angular momentum problem

- **Uniform-density spherical cloud rotating at Ω:**

$$H = I\Omega = \frac{2}{5} MR^2\Omega$$ Angular momentum

$$T = \frac{1}{2} I\Omega^2 = \frac{5H^2}{4MR^2}$$ Rotational KE

$$V = -\frac{3GM^2}{5R}$$ Gravitational binding energy

- **Spherically symmetric collapse possible for initially small values of ratio:**

$$\epsilon = \frac{2T}{|V|} = \frac{25\,H^2}{6G\,M^3\,R}$$

Collapse and flattening

$$\rho \sim \frac{1}{R^3}\,;\,\Omega \sim \frac{1}{R^2} \sim \rho^{2/3}\,;\,\epsilon \sim \frac{1}{R} \sim \rho^{1/3}$$

Isotropic contraction

Angular momentum conserved

Steady increase in ε

- **e.g. for initial conditions:**

Galactic rotation rate

$$\Omega_0 = 10^{-15}\,s^{-1}\,,\,\rho_0 = 10^8\,m_H\,kg\,m^{-3}$$
$$\epsilon_0 \Rightarrow 1/70$$

- **So ε -> 1 after contraction by factor ~ 70 in R**
  - At this stage ρ ~ 3x10¹³ m_H kg m⁻³ << protostellar values.
- **Now, contraction in directions normal to rotation is impeded by centrifugal forces.**
- **System will flatten spontaneously.**

Observed values

- **Typical Ω observed in envelopes of molecular clumps are of order 1 km $s^{-1}pc^{-1}$ = 3x10^{-14} rad s^{-1}.**
- **Slightly faster than Galactic rotation, enhanced by:**
  - turbulence
  - cloud-cloud collisions
- **Similar values observed for NH_3 cores in Taurus-Auriga**
  - No spin-up during contraction
  - Implies outward angular momentum transport during contraction.

    ---> May be due to magnetic braking by Alfven waves.

Conditions at onset of collapse

- **Core mass M_c~ 1 M_{Sun}= 2x10^{30} kg.**
- **Core radius R_c~ 0.1 pc = 3.09x10^{15} m.**
- **Rotation rate Ω ~ 3x10^{-14} rad s^{-1}.**
- **Mean density ρ ~ 10^{10} m_H kg m^{-3}.**
- **Ang. mom. (uniform ρ) J = 2.3x10^{47} kg m^2 s^{-1}**
- **J/M = 1.2x10^{17} m^2 s^{-1}.**
- **If magnetic braking can hold Ω constant until R_c~ 0.03 pc, then outer parts of cloud will have same (j/m) as Keplerian disc material orbiting at 33.2 AU:**

Specific angular momentum
J/M = ΩR²

$$\Omega_{\rm core}R_{\rm core}^2 \approx \Omega_{\rm disc}R_{\rm disc}^2 = \left(GM_{\rm core}R_{\rm disc}\right)^{1/2}$$

$$\Rightarrow R_{\rm disc} \approx \frac{\Omega_{\rm core}^2 R_{\rm core}^4}{GM_{\rm core}}$$

Core material orbiting with Ω_{core} at R_{core}

Keplerian disc material orbiting with Ω_{disc} at R_{disc}

A toy model of collapse

- **Concentric cylinders of cloud have same specific angular momentum.**
- **Mass enclosed inside cylinder is :**

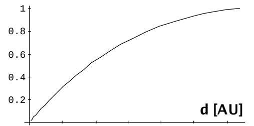
$$m = M_c\left(1 - \left(1 - x^2\right)^{3/2}\right) \text{ where } x \equiv r / R_c$$

- **Each fluid parcel conserves specific angular momentum.**
- **After collapse to disc, each parcel orbits at Keplerian rate for enclosed mass m as above.**
- **Hence orbital radius d in disc of parcel from cylindrical radius r is:**

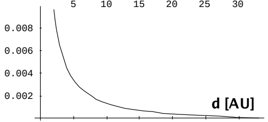
$$d = \frac{\Omega_c^2 r^4}{Gm} = \frac{\Omega_c^2 R_c^4}{GM_c}\,\frac{x^4}{1 - \left(1 - x^2\right)^{3/2}}$$

Central condensation and disc.

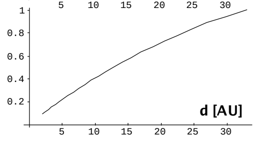
Mass enclosed within distance d [in M_{Sun}].



Surface density of material in disc at distance d [in M_{Sun}(AU)$^{-2}$].



Specific angular momentum of material in disc at distance d [in units of $R^2\Omega$ = 2.58 x10^{16} m^2s^{-1}].



Specific angular momenta

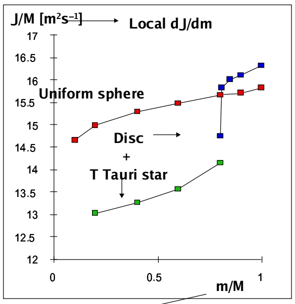
Object	J/M (m^2s^{-1})
Molecular cloud (1 pc)	10^{19}
Molecular cloud core (0.1 pc)	10^{17}
Binary (P=10^{4} yr)	4×10^{16} - 10^{17}
Binary (P=10 yr)	4×10^{15} - 10^{16}
Binary (P=3 day)	4×10^{14} - 10^{15}
TTS (starting contraction)	5×10^{13}
0.5-day rotator on ZAMS	5×10^{12}
7-day rotator on ZAMS	4×10^{11}
Present Sun	10^{11}
Jupiter (orbit)	10^{16}

Numerical simulations of collapse

- Bodenheimer et al 1988, in *Formation and Evolution of Low-Mass Stars*, eds. A K Dupree & M T V T Lago, Kluwer (NATO ASI series).
- Initial conditions:
  - Uniform rotation
  - Spherical cloud
  - Power-law density distribution
  - Isothermal
  - Total mass about 1 M_{Sun}
- Assumptions:
  - No magnetic support
  - Each mass element conserves J
- Results:
  - Rapidly rotating central condensation, radius ~ 1 AU
  - Keplerian disc, radius ~ 100 AU

Redistribution of angular momentum

- Uniformly rotating spherical cloud core.🟥
- T Tauri star🟩 + disc🟦 with same total mass as cloud core (= 1 M_{Sun}) and J.
- TTS has M=0.8 M_{Sun}, R=4R_{Sun}, v_{eq}=50 kms^{-1}.
- Keplerian disc has R_{outer}= 20AU, M=0.2 M_{Sun}.



Mass fraction interior to cylinder concentric with rotation axis.

Rotation of central object

- Conservation of J for each mass element during collapse ->
  - too much spin on central condensation (central core of star within 1AU is very rapidly rotating and unstable to non-axisymmetric perturbations)
  - not enough mass in the middle.
- Need outward angular momentum transport during or after collapse.
- Likely transport mechanisms:
  - Magnetic fields
  - [Turbulent?] shear viscosity.