Supercritical clouds $M_{cl} > M_{cr}$

- Self gravity of a clump can overwhelm magnetic support, even if the fields remain frozen in the fluid.
- In this state, cloud evolution is characterised by magnetically diluted collapse.
- The flow of gas along field lines (i.e. cloud flattening) would result in subregions forming with sizes equal to the vertical dimension.
- Regions would have the same mass/flux ratio (on average) as the entire cloud – hence may also be supercritical

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- Leads to high star-forming efficiency:

Need 20% to 50% efficiency to form bound cluster.

20% to 50% of 270 M_{Sun} pc^{-2} in OB stars gives luminosity density 10^4 to 10^5 L_{Sun} pc^{-2}.

- Trapezium region in Orion contains 4000-10000 stars within a diameter of 5pc
- Only ~10% of stars are believed to have been born in bound clusters
- Total production rate of stars in the Galaxy is thought to be ~ 3-5 M_{sun} yr^{-1}

Subcritical clouds $M_{cl} < M_{cr}$

- Indefinite gravitational collapse (formation of stars) cannot be induced by *any* amount of increased external load (such as external pressure) if Φ is conserved (i.e. flux is frozen-in) since M_{cl}/Φ remains fixed and subcritical.
- Solution: only (molecular) ions are tied to field lines.
- Ambipolar diffusion: field lines can slip through the neutrals, allowing supercritical cores to form.
- Long diffusion timescale means inefficient star formation.

Heating & cooling

- Ionisation fraction in typical regions is very low
- Main heating agent: cosmic-ray ionization occurs at rate

$$\zeta \cong 1 \times 10^{-17}\ \mathrm{s}^{-1}$$

- Main cooling agent: optically thin emission from dust, CO etc
- Equilibrium temperature ~ 10 to 15 K
- Cores become optically thick at $n \geq 10^{16}$ m^{-3}
  - Heat trapped => dense cores warmer
  - Higher T => greater critical mass => more massive stars?

Ionization balance

- Two-body recombinations of charged particles and on charged grains give ion-neutral balance:

$$\rho_i = \underset{\downarrow}{C}\rho_n^{1/2};\ C \approx 9.5 \times 10^{-15}\ \mathrm{m}^{-3/2}\mathrm{kg}^{1/2}$$

Weak function of gas temperature; value given for T ~ 10 to 30 K, metal depletion 0.1

- Typically:

$$\mu_i \approx 30,\ \mu_n \approx 2.3$$

- Hence for n ~ 10^{10} m^{-3}, get ionization fraction

$$\frac{n_i}{n} \approx \frac{C}{\mu_i}\left(\frac{\mu_n}{n m_H}\right)^{1/2} \approx 1.2 \times 10^{-7}$$

- At densities much higher than 10^{14} m^{-3}, natural radioactiviy dominates over cosmic rays as the primary ionising agent

  - Dust grains begin to carry a major fraction of the total charge
  - ρ_i approaches a constant value

Drag force of neutrals

- **Ions tied to field:**

Cyclotron frequency $\frac{eB}{m_i} \gg \gamma \rho_n$ Mean ion-neutral collision frequency = no of collisions per sec felt by ion

Hence ion completes many cycles around fieldline between collisions

NB: $\gamma \approx 3.5 \times 10^{10}\ \mathrm{m^3 kg^{-1} s^{-1}}$ Dipole moment induced in neutral by passing ion

$$\sim \frac{\pi (r_i + r_n)^2 |u_d|}{m_i + m_n}\ \mathrm{m^3 kg^{-1} s^{-1}}$$

Geometric value for collisions with u_d > 10 km s-1

- **Collision between ions and neutrals -> exchange of momentum -> neutrals exert a frictional (or drag) force on the ions.**
- **This force is balanced by the Lorentz force.**

i.e. Exchange of momentum / unit volume

$$\mathbf{j} \times \mathbf{B} = \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} = \rho_i \rho_n \gamma (\mathbf{u}_i - \mathbf{u}_n)$$

i.e. Using Ampères law

Hence the ion-neutral drift velocity $\mathbf{u_d}$

$$\boldsymbol{u}_d \equiv (\boldsymbol{u}_i - \boldsymbol{u}_n) = \frac{1}{\rho_n \rho_i \gamma \mu_0} (\nabla \times \boldsymbol{B}) \times \boldsymbol{B}$$

Ambipolar diffusion

- **Induction equation for the ion fluid**

$$\frac{\partial \boldsymbol{B}}{\partial t} + \nabla \times (\boldsymbol{B} \times \boldsymbol{u}_i) = 0$$

- **Substitute to get nonlinear diffusion equation for neutrals:**

$$\frac{\partial \boldsymbol{B}}{\partial t} + \nabla \times (\boldsymbol{B} \times \boldsymbol{u}_n) = \nabla \times \left\{ \frac{\boldsymbol{B}}{\mu_0 \rho_i \rho_n \gamma} \times [\boldsymbol{B} \times (\nabla \times \boldsymbol{B})] \right\}$$

Effective diffusion coefficient

- **Express RHS in terms of a diffusion coeff:**

$$D \sim u_A^2 t_{ni}, \text{ where } u_A^2 \equiv \frac{B^2}{\mu_0 \rho_n} \text{ and } t_{ni} \equiv \frac{1}{\rho_i \gamma}$$

Alfvén speed in combined medium

Collision time for neutral in sea of ions

- **Diffusion and dynamical timescales for field with length scale R:**

$$t_{AD} \sim \frac{R^2}{D} \sim \frac{R}{u_d} \text{ and } t_{dyn} \sim \frac{R}{u_A}$$

Typical timescales

- **Cloud core of mass 1 M_{Sun}, radius 0.1 pc and critical flux density has:**

$$u_A^2 \sim \frac{GM}{R} \Rightarrow t_{dyn} \sim \frac{R}{u_A} \sim \frac{R^{3/2}}{(GM)^{1/2}}$$

- **This gives a dynamical timescale of a few times 10^5 y, and an ambipolar diffusion time an order of magnitude or so greater.**
- **From detailed analysis in slab geometry (see Shu 1987, *Ann Rev A & A* 25, 23):**

$$\frac{t_{AD}}{t_{dyn}} \sim \frac{\gamma C}{2(2\pi G)^{1/2}} \approx 8$$

- **Hence ambipolar diffusion can be expected to take place at a relatively slow rate until the ionisation fraction begins to depart appreciably from** $\rho_i = C \rho_n^{1/2}$