Supersonic turbulence?

- **If CO linewidths interpreted as turbulence, velocities approach virial values:**

$$\Delta u \sim \left(\frac{2GM}{R} \right)^{1/2}$$

Molecular line FWHM in km/sec

Cloud radius

- **Problem: specific energy ~ u^2 in macroscopic motions of length scale l gets converted to heat (cascading eddies, radiative shocks) in time ~ l/u.**
- **Need u ~ virial speed and l << R for support.**
- **But this means dissipation time l/u << crossing time R/u, so need constant replenishment (winds??)**
- **Ordered magnetic field => turbulence not dominant.**

Magnetic support

- **A fancier form of the scalar virial theorem:**

$$\frac{1}{2}\frac{d^2 I}{dt^2} = 3Mc_s^2 + \Omega$$
$$+ \int_V \left(\frac{B^2}{2\mu_0} \right) dV$$
$$- \int_S \left(p + \frac{B^2}{2\mu_0} \right) \mathbf{r.dS} + \frac{1}{\mu_0} \int_S \mathbf{(r.B)B.dS}$$

Volume integrated magnetic energy

Surface term due to external gas + magnetic pressure at boundary

Surface term due to mag. tension

Contracting spherical cloud

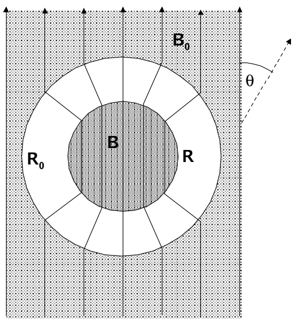
- **Uniform external field B_0 outside R_0.**
- **Uniformly compressed field inside R:**

$$\pi R^2 B = \pi R_0^2 B_0 = \Phi$$

Total flux threading cloud

- **Radial field between R_0 and R:**

$$B(r,\theta) = B_0 \frac{R_0^2}{r^2} \cos\theta$$



Magnetic volume term inside R_0

$$\int_V \left(\frac{B^2}{2\mu_0} \right) dV = \frac{4}{3}\pi R^3 \frac{B^2}{2\mu_0}$$
$$+ \int_0^{\pi} \int_R^{R_0} \frac{[B(r,\theta)]^2}{2\mu_0} .2\pi r \sin\theta . r\, dr d\theta$$
$$= \frac{2\pi}{3\mu_0} B^2 R^3 + \frac{2\pi}{3\mu_0} B_0^2 R_0^4 \left[\frac{1}{R} - \frac{1}{R_0} \right]$$

Energy of uniform field inside inner sphere

Energy of radial field between surfaces R and R_0

Magnetic surface terms at R_0

$$S = \frac{1}{\mu_0} \int_S \mathbf{(r.B)B.dS} - \int_S \frac{B^2}{2\mu_0} \mathbf{r.dS}$$

- **Can compute surface integral over R_0 directly, but beware of sign errors!**
- **More simply, note that when R= R_0 , field is uniform and so exerts no forces and makes no net contribution to Virial Theorem:**

$$\frac{2\pi}{3\mu_0} B_0^2 R_0^3 + \frac{2\pi}{3\mu_0} B_0^2 R_0^4 \left[\frac{1}{R_0} - \frac{1}{R_0} \right] + S = 0$$

- **Hence deduce that volume and surface terms add to zero when R= R_0 , so the sum of the two surface terms must be:**

$$S = -\frac{2\pi}{3\mu_0} B_0^2 R_0^3$$

Total magnetic virial contribution

- **Sum of volume terms:**

$$\frac{2\pi}{3\mu_0} B^2 R^3 + \frac{2\pi}{3\mu_0} B_0^2 R_0^4 \left[\frac{1}{R} - \frac{1}{R_0} \right]$$

- **Sum of surface terms:**

$$-\frac{2\pi}{3\mu_0} B_0^2 R_0^3$$

- **Total (volume + surface) for R < R_0 is:**

$$\frac{4\pi}{3\mu_0} B_0^2 R_0^4 \left[\frac{1}{R} - \frac{1}{R_0} \right]$$

Magnetic support

- Add surface and volume terms, and balance against gravitational binding energy for uniform sphere:

$$\frac{4\pi}{3\mu_0} B_0^2 R_0^4 \left[\frac{1}{R} - \frac{1}{R_0}\right] = \frac{3GM^2}{5R}$$

$$\Rightarrow 1 - \frac{R}{R_0} = \frac{9\pi\mu_0 G M^2}{20\Phi^2}$$

ie magnetic energy is sufficient to support cloud at radius R.

- Set R=0 to get minimum flux/mass needed to prevent collapse:

$$\frac{\Phi}{M} = \frac{3(\pi\mu_0)^{1/2} G^{1/2}}{\sqrt{20}} \quad (1)$$

Alfvén speed in critical-mass clouds

- For clouds near critical state, can rearrange condition for support to get:

$$\frac{B^2}{\mu_0 \rho} = \frac{3}{5}\frac{GM}{R} \Rightarrow M u_A^2 = \frac{3}{5}\frac{GM^2}{R}$$

i.e. Alfvén speed is automatically of the magnitude needed for virial equilibrium.

- May explain non-thermal velocities as Alfvén-wave motions.
- Nonlinear Alfvén waves (i.e. in the weak field regime, two Alfvén waves can couple to give an acoustic wave) may provide support against collapse along field into sheets.

Reality check

- Critical mass for clouds with typical observed R, B (get B from Zeeman splitting of OH lines):

$$M_{\rm cr} = 750 \frac{\pi R^2 B}{G^{1/2}} = 1.66 \times 10^3 M_{\rm Sun} \left(\frac{B}{3\rm nT}\right)\left(\frac{R}{2\rm pc}\right)^2$$

- So, clouds with constant B, with clumps near the critical state, $M_{cl} = M_{cr}$ (i.e. clouds where magnetic fields provide the dominant means of support) should have nearly the same mean column densities:

$$\frac{M_{cr}}{\pi R^2} \propto \rho R \sim \text{constant}$$

And since

$$u_A \propto \rho^{-1/2} \longrightarrow u_A \propto R^{1/2}$$

Possible explanation of observed power-law relation between CO linewidth and clump radius

Cloud contraction and fragmentation

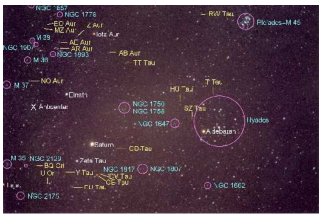
- Critical surface density and visual extinction for flattened critical-mass clouds:

$$\frac{M_{\rm cl}}{\pi R^2} > 270 \frac{M_{\rm Sun}}{\rm pc^2}\left(\frac{B}{3\rm nT}\right)$$

$$\Rightarrow A_V > (4 \text{ to } 5 \text{ mag}) + 2.5 \log\left(\frac{B}{3\rm nT}\right)$$

(typical gas/dust ratios)

- cf. Observed values:
  - Taurus-Auriga: A_V ~ 1-2 mag in envelope, ~ 10 mag in cores with T ~ 10 to 11 K. Subcritical, low efficiency formation of low-mass stars in unbound cluster



  - ρ Oph: A_V ~ 6 mag in envelope, ~ 10^2 mag in densest cores with T ~ 30 to 35 K. Supercritical, high efficiency formation of low-mass stars + a few B stars in bound cluster