The story so far:

Induction

$$u_\mathrm{p} = \kappa B_\mathrm{p}$$

$$\frac{u_\phi}{R} - \frac{\kappa B_\phi}{R} = \Omega = \text{constant}. \quad (1)$$

Mass continuity

$$\rho \frac{u_\mathrm{p}}{B_\mathrm{p}} = \rho\kappa \equiv \eta = \text{constant}. \quad (2)$$

Torque balance

$$\rho\kappa R u_\phi - \frac{R B_\phi}{\mu_0} \equiv \rho\kappa L = \text{constant}. \quad (3)$$

The Alfvénic point

- **Use (1) - $\mu_0\kappa/R^2$ (3) to eliminate B_ϕ:**

$$\frac{u_\phi}{R} - \frac{\mu_0\rho\kappa^2 u_\phi}{R} = \Omega - \frac{\mu_0\rho\kappa^2 L}{R^2}$$

- **Substitute:** $\kappa^2 = \dfrac{u_\mathrm{p}^2}{B_\mathrm{p}^2}$ and $u_\mathrm{A}^2 = \dfrac{B_\mathrm{p}^2}{\mu_0\rho}$,
- **to get:** $\dfrac{u_\phi}{R}\left(1 - \dfrac{u_\mathrm{p}^2}{u_\mathrm{A}^2}\right) = \Omega - \dfrac{u_\mathrm{p}^2 L}{u_\mathrm{A}^2 R^2}.$
- **u_ϕ is singular at Alfvenic point $u_\mathrm{p}=u_\mathrm{A}$ unless:**

$$R_\mathrm{A}^2\Omega - L = 0.$$

Weber-Davis radial-field model

- **Radial field lines close to star ("split monopole").**
- **Spherical Alfvénic surface, radius r_A.**
- **Isotropic mass loss.**
- **Angular momentum flux across area ds is:**

$$(\rho_\mathrm{A} u_\mathrm{A}).\left(\Omega r_\mathrm{A}^2 \cos^2\theta\right).\left(2\pi r_\mathrm{A}\cos\theta . r_\mathrm{A} d\theta\right)$$

Mass flux — Net specific angular momentum transported along field line anchored at latitude θ — Area of surface element ds

$$\Rightarrow -\dot{J} = \left(\rho_\mathrm{A} u_\mathrm{A} r_\mathrm{A}^2\right)\left(\Omega r_\mathrm{A}^2\right) 2\pi \underbrace{\int_{-\pi/2}^{\pi/2} \cos^3\theta\, d\theta}_{=4/3}$$

Net spindown torque on star

- **Density at Alfven radius:** $\rho_\mathrm{A} = \dfrac{B_\mathrm{A}^2}{\mu_0 u_\mathrm{A}^2}$
- **Net torque on star:**

$$\Rightarrow -\dot{J} = \frac{8\pi}{3\mu_0}\left(B_\mathrm{A} r_\mathrm{A}^2\right)^2 \frac{\Omega}{u_\mathrm{A}} = \frac{8\pi}{3\mu_0}\left(B_0 r_*^2\right)^2 \frac{\Omega}{u_\mathrm{A}}$$

Radial or dipole field

- **For thermal driving, $u_\mathrm{A}\sim$ 2 to 3 c_s, indep. of Ω.**
- **For linear dynamo law, $B_0 \sim \Omega$.**
- **If stellar moment of inertia is constant:**

$$J = k^2 M_* r_*^2 \Omega \Rightarrow \boxed{\frac{d\Omega}{dt} \propto -\Omega^3}$$

General braking laws

$$\frac{d\Omega}{dt} = -c_0\Omega^p \Rightarrow \frac{1}{1-p}\left(\Omega^{1-p} - \Omega_0^{1-p}\right) = -c_0(t - t_0)$$

$$\Rightarrow \Omega = \left[\Omega_0^{1-p} + c_0(p-1)(t-t_0)\right]^{1/(1-p)}$$

- **Asymptotically:**

$$\Omega \to c_0(p-1)(t-t_0)^{1/(1-p)} \quad \text{for } t - t_0 >> \frac{\Omega_0^{1-p}}{c_0(p-1)}$$

- **Hence for p = 3,**

$$\Omega(t) \to t^{-1/2}$$

cf. Skumanich.