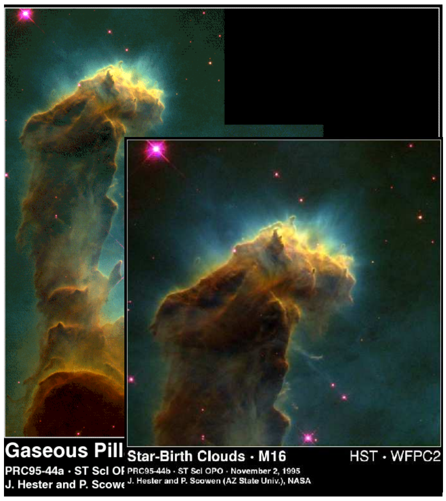
MOLECULAR CLOUDS

- **Giant molecular clouds – CO emission**
  - several tens of pc across
  - mass range 10^5 to 3×10^6 M_o
  - clumpy substructure
  - lifetimes ~ 10^7 y ~ crossing time of clumps
- **Temperatures too cold for H_2 and He to emit**
- **Trace molecules like CO, H_2O, HCN, NH_3 excited by collisions with H_2**
- **Several thousand radiative transitions in range 0.7 GHz (43 cm) to 3800 GHz (77 μm).**
- **CO 10^4 times less abundant than H_2**
- **Others rarer still.**
- **Some density diagnostics:**
  - excitation of CO requires $n(H_2) \geq 10^8$ m^{-3}
  - excitation of NH_3 requires $n(H_2) > 10^{10}$ m^{-3}

- **Substructures (clumps) – CO emission**
  - M_{cl} ~ 10^3 to 10^4 M_o
  - R ~ 2 to 5 pc
  - $n(H_2)$ ~ $10^{8.5}$ m^{-3}
  - T ~ 10 K
  - cf. Taurus - Auriga complex.
- **Cores – NH_3, H_2CO, HC_3N, CS emission**
  - M_{core} ~ 1 M_o; massive envelope ~ 10^2 M_o
  - R ~ 10^{-1} pc
  - $n(H_2) > 10^{10}$ m^{-3}
  - T ~ 10 K



Scalar virial theorem

- **Start with a set of particles.**
- **Stationary wrt an inertial frame at time t_0.**
- **Typical particle: mass m, position <u>r</u>(t) acted on by force <u>P</u> has eq. of motion:**

$$m\ddot{r} = P$$

$$\frac{d^2}{dt^2}(\mathbf{r}.\mathbf{r}) = 2\frac{d}{dt}(\mathbf{r}.\dot{\mathbf{r}}) = 2\dot{\mathbf{r}}^2 + 2\mathbf{r}.\ddot{\mathbf{r}}$$

Sum over all particles

$$\frac{1}{2}\frac{d^2}{dt^2}\left(\sum m\mathbf{r}^2\right) = \sum m\dot{\mathbf{r}}^2 + \sum \mathbf{r}.\mathbf{P}$$

Moment of inertia I — Twice total thermal KE of system: — "Virial"

Temperature, assumed uniform

$$2M\times\frac{3kT}{2\mu m_H} = 3Mc_s^2 \text{ where } c_s^2 = \frac{kT}{\mu m_H}$$

Total mass of particles — Mean mol. wt. — Mass of H atom — Isothermal sound speed

The virial

- **Forces <u>P</u> contributing to virial at points of application <u>r</u>:**
  - collisions with other particles in system
  - self-gravitation due to other particles in system
  - collisions with external material

Equal & opposite pairs, so no net contribution

$$\int_V \mathbf{r}.\mathbf{g}\rho dv = \Omega \equiv -A\frac{GM^2}{R}$$

Grav. force per unit mass at <u>r</u> — Mass of particles in vol. element dv at <u>r</u> — Grav. binding energy

Produce pressure P at external boundary S, contributing

$$\int_S p\mathbf{r}.d\mathbf{S} = -p\int_V \nabla.\mathbf{r}dv = -3pV$$

Inward normal — (For p uniform over S)

Virial equilibrium

$$\frac{dI}{dt} = 0 \quad (t = t_0)$$

- **For a body of gas released from rest at time t_0 under given external pressure p:**

$$\frac{1}{2}\frac{d^2I}{dt^2} = 3Mc_s^2 - 3pV + \Omega$$

- **If the initial state is also an equilibrium state we must also have:**

$$\frac{d^2I}{dt^2} = 0 \quad (t = t_0)$$

> 0 gives expansion

< 0 gives contraction

Self gravitation & thermal pressure

- **Scalar virial equilibrium – pure thermal support:**

$$p_0 V = Mc_s^2 - A\frac{GM^2}{3R}$$

Thermal pressure of warm surrounding ISM (p_0)

Volume: for sphere,

$$V = \frac{4\pi R^3}{3}$$

Virial coefficient A=(3/5) for uniform sphere, increasing with central condensation

Spherical cloud

- **Equilibrium condition becomes:**

$$p_0 = \frac{3Mc_s^2}{4\pi R^3} - A\frac{GM^2}{4\pi R^4}$$

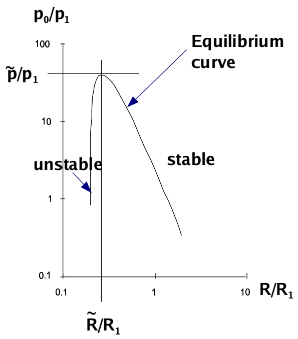
$$\text{Define } R_1 = \frac{GM}{c_s^2},\ p_1 = \frac{c_s^8}{4\pi G^3 M^2},$$

$$\Rightarrow \frac{p_0}{p_1} = 3\left(\frac{R_1}{R}\right)^3 - A\left(\frac{R_1}{R}\right)^4.$$

Maximum equilibrium pressure

- **For fixed M, c_s, get p-R relation defined by equilibrium condition:**
- **2 equilibria: one stable, other unstable.**
- **No equilibrium possible for pressures greater than point on relation where $dp_0/dR = 0$:**

$$\frac{\tilde{R}}{R_1} = \frac{4}{9}A, \quad \frac{\tilde{p}}{p_1} \sim \frac{3}{4}\left(\frac{4}{9}A\right)^{-3}$$



Jeans mass & length

- **Express critical R, M in terms of mean density:**

$$\tilde{R} = \frac{4AGM}{9c_s^2} = \frac{4AG}{9c_s^2}\cdot\frac{4\pi\tilde{R}^3\bar{\rho}}{3}$$

$$\Rightarrow \tilde{R} = \left[\frac{27}{16\pi A}\cdot\frac{c_s^2}{G\bar{\rho}}\right]^{1/2} \sim \text{Jeans length } \lambda_J$$

$$\tilde{M} = \frac{4\pi\tilde{R}^3\bar{\rho}}{3}$$

$$= \frac{4\pi}{3}\left[\frac{27}{16\pi A}\right]^{3/2}\left[\cdot\frac{c_s^2}{G}\right]^{3/2}\left[\frac{1}{\bar{\rho}}\right]^{1/2} \sim \text{Jeans mass } M_J$$

Non-thermal support

- **Using these expressions, for average clump $n=10^{8.5}$ m^{-3} and T=10K:**
  - **We find that $M_J \sim$ few $M_\odot$, $\lambda_J \sim 1$ pc**
- **But clumps have masses 10^3 to 10^4 $M_\odot$ and are NOT collapsing on a free-fall timescale.**

Clumps are much larger than the critical Jeans Mass, but are not undergoing free-fall collapse

- **Need some other means of support.**
- **Two main observational clues:**
  - **High CO linewidths Δu imply very supersonic fluid motion**
  - **Polarization maps indicate ordered magnetic fields**
- **Empirical power-law relation:**

$$\Delta u \propto R^{\alpha},\ \alpha \approx 0.5$$