## Balancing heating with losses

From the **energy equation,** if there is **no Ohmic heating,**

$$\rho\frac{De}{Dt}+p\nabla.\underline{v}=-L=-\nabla.\underline{q}-L_r+H$$

Radiation

Heat flux

Other heating sources – e.g. viscous dissipation, wave heating

If the plasma is **thermally isolated** there is no exchange of heat and L = 0. This is the **adiabatic** case.

$$H=\nabla.\underline{q}+L_r$$

The heat flux **q** can be written in terms of the conductivity $\kappa$

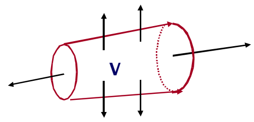
$$q=-\kappa\nabla T$$

(ie heat flows in the direction of decreasing temperature)



- Now the rate of heat loss from a flux tube of volume V can also be written as the heat flux through the surface:

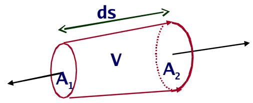
$$\int\int\int_V\nabla.\underline{q}=\int\int_s\underline{q}.dS$$




**BUT**: we only need to consider conduction **along** B, ie the flux through the two ends of the flux tube

Hence the rate of heat loss **per unit volume** is

$$\nabla.q\approx\frac{q_2A_2-q_1A_1}{A\,ds}\approx\frac{d(qA)}{A\,ds}$$






i.e. $$-\nabla_{\parallel}.(\kappa_{\parallel}\nabla_{\parallel}T)=-\frac{1}{A}\frac{d}{ds}\left(\kappa_{\parallel}\frac{dT}{ds}A\right)$$

where $$\kappa_{\parallel}=10^{-11}T^{5/2}=\kappa_0T^{5/2}\,W\,M^{-1}\,deg^{-1}$$

chromosphere / corona

For an optically thin plasma, the **radiative losses** depend on the **radiative loss function Q(T) Wm³** (see handout)

$$L_r=n_en_HQ(T)=n_en_H\chi T^{\alpha}$$

We can write this in terms of the density $\rho$ = **mn** where for a fully-ionised H plasma,the total particle number **n=2n$_e$** and the mean particle mass **m=0.6m$_p$** (for the solar atmosphere)

i.e. $L_r=\rho^2\tilde{\chi}T^{\alpha}$ where $\tilde{\chi}=\chi/4m^2$



## Applying this to a magnetic loop

Hence, for a **static** loop in thermal equilibrium with $\alpha$ = -1/2

$$H=n_en_H\chi T^{-1/2}-\frac{1}{A}\frac{d}{ds}\left(\kappa_0T^{5/2}\frac{dT}{ds}A\right)$$

where $p=(n_e+n_i)k_BT\approx2n_ek_BT$

For very low-lying (**uniform pressure**) loop with a **uniform A** and summit temperature **T$_s$**, then since globally radiation and conduction are similar in magnitude,

$$\frac{p^2}{4k_B^2}\chi T_s^{-5/2}\approx\kappa_0\frac{T_s^{7/2}}{L^2}$$ i.e. $$T_s\propto(Lp)^{1/3}$$



Also, because globally **heating** and **radiation** are of the same order,

$$H\approx\frac{p^2}{4k_B^2}\chi T_s^{-5/2}$$

i.e. $$H\propto L^{-5/6}p^{7/6}$$

If we consider the heating to be specified, we may combine these to give:

$$p\propto H^{6/7}L^{5/7}$$ and $$T_s\propto H^{2/7}L^{4/7}$$

Both **p** and **T** increase when heating **H** increases or when the loop is stretched (**L** increases).



## Thermal Instability

We can rewrite the energy equation as

$$\rho\frac{D}{Dt}(c_pT)-\frac{Dp}{Dt}=-L$$

using $c_v=k_B/m(\gamma-1)$ and $e=c_vT=c_pT-\frac{p}{\rho}$

Hence if **the pressure remains constant**

$$\rho\frac{D}{Dt}(c_pT)=-L$$

This describes how the temperature changes in response to an imbalance in L.

**Start with a plasma in equilibrium:**

A plasma has temperature $T_0$ and density $\rho_0$ under a balance between radiation and heating per unit volume ($H = h\rho$) where h is a constant.

Per unit mass $h = \tilde{X}\rho_0 T_0^\alpha$

Now perturb this system at constant pressure to find a new temperature and density:

$$c_p \frac{\partial T}{\partial t} = h - \tilde{X}\rho T^\alpha$$

Note that since perturbation is small, $DT/Dt \rightarrow \partial T/\partial t$



Since the pressure is constant, the new density is

$$\rho = \frac{m p_0}{k_B T} = \rho_0 \frac{T_0}{T}$$

And so we have

$$c_p \frac{\partial T}{\partial t} = \tilde{X}\rho_o T_0^\alpha \left(1 - \frac{T^{\alpha-1}}{T_0^{\alpha-1}}\right)$$

$O(c_p T_0/\tau)$ $O(\tilde{X}\rho_0 T_0^\alpha)$



Hence if $\alpha < 1$ a small decrease in T ($T < T_0$)
=> RHS < 0
=> the perturbation continues since

$$\partial T/\partial t < 0$$

This thermal instability has a timescale

$$\tau_{rad} = c_p/\tilde{X}\rho_0 T_0^{\alpha-1}$$



This instability can be opposed by the effect of conduction which would add an extra term to the perturbed energy equation

$$c_p \frac{\partial T}{\partial t} = h - \tilde{X}\rho T^\alpha + \rho^{-1}\underline{\nabla}.(\kappa_0 T^{5/2}\underline{\nabla} T)$$

$O(c_p T_0/\tau)$ $O(\kappa_0 T_0^{7/2}/\rho L^2)$

For a fieldline of length L, the conduction time is

$$\tau_c = L^2 \rho_0 c_p/\kappa_0 T_o^{5/2}$$



Now, if L is small enough that $\tau_c < \tau_{rad}$

then conduction can prevent the runaway drop in temperature and the plasma is thermally stable.

However, there is a maximum loop length (where the two timescales are equal) given by

$$L_{max} = (\kappa_0 T_0^{7/2-\alpha}/\tilde{X}\rho_0^2)^{1/2}$$

Loops longer than this may be thermally unstable.



## Observing stellar coronae

For an isothermal plasma at a temperature T, the power in a line radiated from a volume V is

$$P = \beta G(T)\int n_e^2 dV$$

$\beta$ contains atomic parameters and abundancies

$G(T)$ the contribution function is strongly peaked in T

$\int n_e^2 dV$ the emission measure can be calculated from P if the temperature is known. Note that this is often used interchangeably with the X-ray flux.

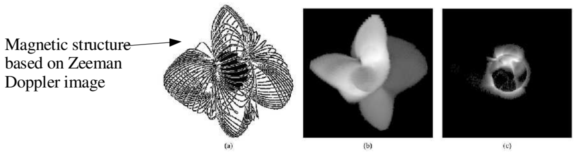



**Figure 5.** The effect of the coronal temperature on the emission measure. Part (a) shows the magnetic structure of the large-scale field based on the surface map shown in Fig. 1. The field is viewed from longitude 128°. Parts (b) and (c) show the corresponding emission measure images on a square root stretch at temperatures of $10^7$ and $10^6$ K, respectively. The corresponding emission-measure-weighted densities are $4 \times 10^8$ and $2 \times 10^8$ cm$^{-3}$.

**Problems:**

- **The temperature and density cannot be found independently. Often only a temperature range can be specified.**
- **Instrumental response must be taken into account when converting X-ray flux to emission measure.**
- **Several temperature components may contribute (unequally) to the observed emission in any temperature range.**