Magnetic Forces

The **Lorentz** force

$$F_m = \underline{j}\times\underline{B} = (\underline{\nabla}\times\underline{B})\times B/\mu$$

can be rewritten using the vector identity

$$(\underline{\nabla}\times\underline{B})\times B = -\underline{\nabla}B^2 + (B.\underline{\nabla})\underline{B} + (\underline{B}.\underline{\nabla})\underline{B} + \underline{B}\times(\underline{\nabla}\times\underline{B})$$

to give

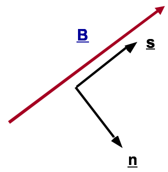
$$F_m = \underbrace{(\underline{B}.\underline{\nabla})\underline{B}/\mu}_{\text{tension}} - \underbrace{\nabla(B^2/2\mu)}_{\text{pressure}}$$

We can decompose the Lorentz force into components **along** and **across** B. The tension term gives, with $\underline{B} = B\underline{\hat{s}}$

$$(\underline{B}.\underline{\nabla})\frac{\underline{B}}{\mu} = B\frac{\partial}{\partial s}\left(\frac{B\underline{\hat{s}}}{\mu}\right)$$

$$= \frac{B}{\mu}\frac{\partial B}{\partial s}\underline{\hat{s}} + \frac{B^2}{\mu}\frac{\partial\underline{\hat{s}}}{\partial s}$$

$$= \frac{\partial}{\partial s}\left(\frac{B^2}{2\mu}\right)\underline{\hat{s}} + \frac{B^2}{\mu}\frac{\underline{\hat{n}}}{R_c}$$

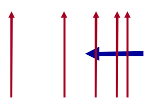


where R_c is the **radius of curvature** of the field. Thus the tension term has components along and across the field.

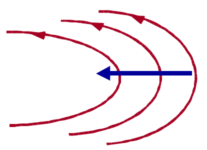
The **pressure** term is $-\nabla\left(\frac{B^2}{2\mu}\right) = -\left(\frac{\partial}{\partial s},\frac{\partial}{\partial n}\right)\frac{B^2}{2\mu}$

Combining the two shows that the components **along** the field cancel, leaving only a term **normal** to the field

$$\underline{j}\times\underline{B} = \left[\frac{B^2}{\mu R_c} - \frac{\partial}{\partial n}\left(\frac{B^2}{2\mu}\right)\right]\underline{\hat{n}}$$



$\nabla\left(\frac{B^2}{2\mu}\right)$

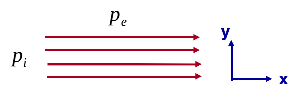


$\frac{B^2}{\mu R_c}$

Zero tension **Zero pressure gradient**

Buoyancy

Simple case: isolated flux tube with straight field lines.



$\underline{B} = (B_x(y), 0)$

$$0 = -\underline{\nabla}p + (\underline{B}.\underline{\nabla})B/\mu - \nabla(B^2/2\mu)$$

The equation of motion reduces to **pressure balance**.

$$p_e = p_i + \frac{B_i^2}{2\mu}$$

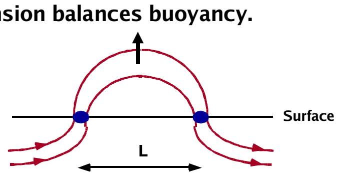
Clearly $p_i < p_e$ and so for an **isothermal** plasma with

$$p = \rho k_B T/m$$

where m ~ 0.6 m_H (mean particle mass) we have

$$\rho_i < \rho_e$$

and so the flux tube is **buoyant**. Flux tubes formed in the convection zone rise under gravity to emerge at the surface forming spot groups. They continue to rise until tension balances buoyancy.



Rough estimate of final loop shape:

The **buoyancy** force / unit volume is: $(\rho_e - \rho_i)g$

The **magnetic tension** force / unit volume is: $B^2/\mu L$

Rise implies $B^2/\mu L < (\rho_e - \rho_i)g$

or $2(p_e - p_i)/L < (\rho_e - \rho_i)g$

which is just $\frac{2k_BT}{Lm}(\rho_e - \rho_i) < (\rho_e - \rho_i)g$

The flux tube will rise if its footpoints are separated by more than twice the pressure scale height, Λ

$$L > \frac{2 k_B T}{m g} = 2\Lambda$$

$\Lambda \sim$ 500km at the solar surface and is calculated from hydrostatic equilibrium.

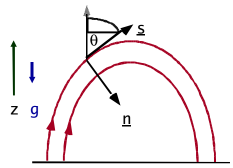
$$\underline{\nabla} p = \rho \underline{g} = \frac{p m \underline{g}}{k_B T} = \frac{p}{\Lambda}$$

$$p = p_0 e^{-z/\Lambda}$$

Pressure structure in the corona

Consider a flux tube anchored in the equatorial plane of a rotating star. Force balance gives

$$\underline{\nabla} p = \underline{j} \times \underline{B} + \rho \underline{g}$$



Component along the field: $\frac{dp}{ds} = -\rho g \cos\theta$

We can rewrite this using $\delta s \cos\theta = \delta z$

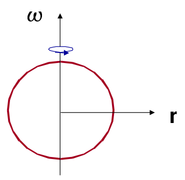
to give (where we now have a local scale height) $p = p_0 e^{-\int_0^z \frac{dZ}{\Lambda(Z)}}$

Remember that in a co-rotating frame of reference

$$\underline{g}_T = \underline{g} - \underbrace{2\underline{\omega} \times \underline{\dot{r}}}_{\text{Coriolis}} - \underbrace{\underline{\omega} \times (\underline{\omega} \times \underline{r})}_{\text{Centrifugal}}$$

ω :angular velocity (rad/s)

In the absence of flows, the Coriolis acceleration is zero and the centrifugal acceleration can be written as



$$-\underline{\omega} \times (\underline{\omega} \times \underline{r}) = -(\underline{\omega} . \underline{r})\underline{\omega} + \omega^2 \underline{r}$$

zero at the equator

So that at the equator

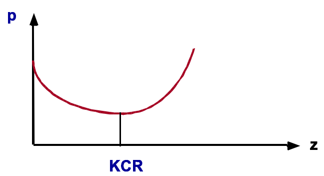
$$\underline{g}_T(z) = \frac{-GM_*}{z^2} + \omega^2 z$$

and $$\Lambda(z) = \frac{k_B T(z)}{m g(z)}$$

For the Sun, the scale height is often taken as constant, which assumes a uniform temperature and constant g (ok for low-lying structures).

For a rotating star, the pressure first falls, then rises at the co-rotation radius (KCR) where gravity balances centrifugal forces and

$$g_T(z) = 0$$

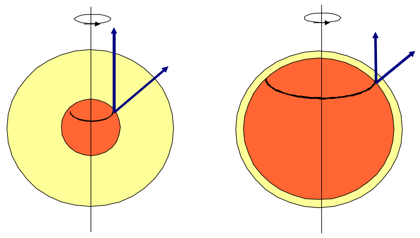


Hence a rapidly-rotating star acts like a centrifuge, driving material into the tops of closed magnetic loops.

Flux tube emergence on rapid rotators

- Flux tubes formed at the base of the convective region rise bouyantly to the surface.
- They rise at the Alfven speed, $v_A = B / (\mu \rho)^{1/2}$ - too rapidly to exchange angular momentum with their surroundings.
- Conservation of angular momentum forces them to rise parallel to the rotation axis.
- Latitude at which they emerge depends on their initial distance from the rotation axis, i.e. the depth of the convection zone.

- For the same rotation rate, stars with deeper convection zones (later spectral types, further down the main sequence) should show higher-latitude spots.
- NB: fully convective stars also show coronae, flares etc…this process does not work for them!



- For a given convection zone depth, latitude of emergence depends on whether buoyancy (-> radial emergence) or Coriolis forces (-> vertical emergence) dominates.
- Ratio of forces:

$$\frac{F_{Cor}}{F_{Buoy}} = \frac{2\rho v_A \omega}{(\rho_e - \rho_i) g} = \frac{2\rho v_A \omega}{(p_e - p_i)/\Lambda} = \frac{4\mu\rho v_A \omega \Lambda}{B_i^2}$$

$$\frac{F_{Cor}}{F_{Buoy}} = \frac{4\omega\Lambda}{v_A}$$

- For a given rotation rate, Coriolis forces dominate and flux tubes are deflected to high latitudes if

$$B < B_m \equiv 4(\mu\rho)^{1/2}\omega\Lambda$$

- For the Sun, B_m is about 10^5 G.

Force-free fields

For a static atmosphere $\nabla p = \underline{j} \times \underline{B} + \rho \underline{g}$

If a) the height of the magnetic structure << the scale height and
b) the plasma pressure << the magnetic pressure (small plasma beta) i.e.

$$\beta = \frac{2\mu p}{B^2} \ll 1$$

then
a) there can be no Lorentz force and
b) the current must flow along field lines.

$$\underline{j} \times \underline{B} \approx 0 \longrightarrow \underline{j} = \alpha(\underline{r}, t)\underline{B}$$

$$\underline{j} = 0 \longrightarrow$$ potential field