Currents and Collisions

The current $j=q(Z n_i v_i - n_e v_e)$

is zero if there is no net drift of the electrons with respect to the ions. This can be achieved if collisions are 100% efficient in randomising particle motions.

The conductivity, σ is a measure of how ineffective collisions are at maintaining this balance.

By definition, σ relates the current *j* produced by a collisional rate of exchange of momentum *P*.

$$\underline{P}_{ei}=\frac{n_e q}{\sigma}\underline{j}$$

High conductivity -> ineffective collisions -> high current

If we put $\underline{P}_{ei}=n_e m_e \underline{v}_e \nu_c$ and $\underline{j}=n_e q \underline{v}_e$ where ν_c is the frequency of collision of electrons with stationary ions, we find that the conductivity

$$\sigma=\frac{n_e q^2}{m_e \nu_c}=\sigma_0$$

If we consider conductivities parallel to and perpendicular to the magnetic field, we find that

$$\sigma_{\parallel}=\sigma_0 \qquad \sigma_{\perp}=\frac{\sigma_0}{1+(\Omega/\nu_c)^2}$$

Instead of the conductivity, we often use the diffusivity

$$\eta=\frac{1}{\mu\sigma}=10^9 T^{-3/2} m^2 s^{-1}$$

Ohm's Law

While adding the equations of motion for electrons and ions gives the behaviour of the total velocity, subtracting the separate equations gives the behaviour of the current.

$$n_e m_e \frac{d\underline{v}_e}{dt}=-n_e q(\underline{E}+\underline{v}_e\times\underline{B})+n_e m_e \underline{g}-\underline{\nabla}p_e+\frac{n_e q}{\sigma}\underline{j}$$

$$n_i m_i \frac{d\underline{v}_i}{dt}=Z n_i q(\underline{E}+\underline{v}_i\times\underline{B})+n_i m_i \underline{g}-\underline{\nabla}p_i-\frac{Z n_i q}{\sigma}\underline{j}$$

To give in a steady state $\partial\underline{j}/\partial t=0$

$$\boxed{\underline{j}=\sigma(\underline{E}+\underline{v}\times\underline{B})}$$

The Induction Equation

Using Ohm's Law and Maxwell's equations gives

$$\frac{\partial\underline{B}}{\partial t}=-\underline{\nabla}\times\underline{E}=\underline{\nabla}\times(\underline{v}\times\underline{B})-\underline{\nabla}\times\underline{j}/\sigma$$

$$\text{i.e. } E=\frac{j}{\sigma}-v\times B$$

Using $\underline{j}=\underline{\nabla}\times\underline{B}/\mu$ (i.e. Maxwell IV)

and $\underline{\nabla}\times(\underline{\nabla}\times\underline{B})=\underline{\nabla}(\underbrace{\cancel{\underline{\nabla}.\underline{B}}}_{0})-(\underline{\nabla}.\underline{\nabla})B$

gives

$$\boxed{\frac{\partial\underline{B}}{\partial t}=\underbrace{\underline{\nabla}\times(\underline{v}\times\underline{B})}_{\text{Advection (field lines carried by flow)}}+\underbrace{\eta\nabla^2\underline{B}}_{\text{Diffusion (energy loss term)}}}$$

The Energy Equation

From the second law of thermodynamics

$$T\,ds=\underbrace{dQ}_{\text{Heat Change}}=\underbrace{dU}_{\text{Internal Energy Change}}+\underbrace{p\,dV}_{\text{Work done}}$$

Using $e=\frac{p}{(\gamma-1)\rho}$ (internal energy / unit mass)

$\gamma=c_p/c_v$ (ratio of specific heats)

$$\frac{D}{Dt}=\left(\frac{\partial}{\partial t}+v.\underline{\nabla}\right)$$

s = entropy / unit mass

L = sum of sources + sinks (energy loss function)

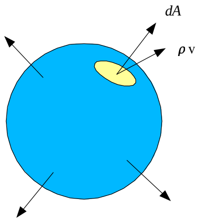
We obtain

$$\rho T\frac{Ds}{Dt}=\rho\left[\frac{De}{Dt}+p\frac{D}{Dt}\left(\frac{1}{\rho}\right)\right]=-L$$

We can write this more simply using mass conservation

$$\frac{\partial\rho}{\partial t}+\underline{\nabla}.\rho\underline{v}=0$$

Continuity equation



Mass continuity means that : the rate of increase of mass inside the volume + rate at which mass is flowing out across the surface = 0

i.e. $\frac{\partial}{\partial t}\left\{\int_V \rho\, dV\right\} + \int_A \rho\, \mathrm{v}\,.\, dA = 0$

Using $\int_A Q\, dA = \int_V \underline{\nabla}.Q\, dV$

$$\int_V \left\{\frac{\partial \rho}{\partial t} + \underline{\nabla}.(\rho\, \underline{v})\right\} dV = 0 \xrightarrow[\text{all points}]{\text{true at}} \frac{\partial \rho}{\partial t} + \underline{\nabla}.(\rho\, \underline{v}) = 0$$

$$\frac{\partial \rho}{\partial t} + \underline{v}.\underline{\nabla}\rho + \rho\underline{\nabla}.\underline{v} = 0 \longrightarrow \boxed{\frac{D\rho}{Dt} = -\rho\underline{\nabla}.\underline{v}}$$

to give
$$\rho\frac{De}{Dt} + p\underline{\nabla}.\underline{v} = -L$$

$$= -\underline{\nabla}.\underline{q} - L_r + j^2/\sigma + H$$

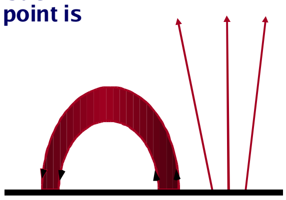
with

$\underline{\nabla}.\underline{q}$	heat flux due to conduction
L_r	net radiation
j^2/σ	Ohmic heating
H	sum of all other heating sources

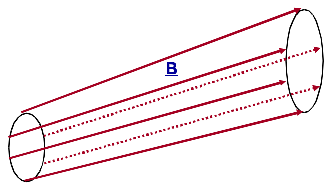
Magnetic Structures

- These can be **closed loops** (trapped plasma) or **open field lines** (plasma escapes as stellar wind).
- Soft X-ray images of the Sun show **closed** regions as **bright**, **open** regions (coronal holes) as **dark**.
- A **magnetic field line** is such that the tangent at any point is in the direction of B. In **cartesians**:

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z}$$



A **magnetic flux tube** is the volume enclosed by the set of field lines that intersect a simple closed curve.



Flux tubes contain plasma (and insulate it from its surroundings) by inhibiting particle motion across field lines.