

# RADIO INTERFEROMETRY

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# Holy Books of the World

Buddhism: Tipitaka

Christianity: Bible

Hinduism: Bhagavad Gita

Islam: Kuran

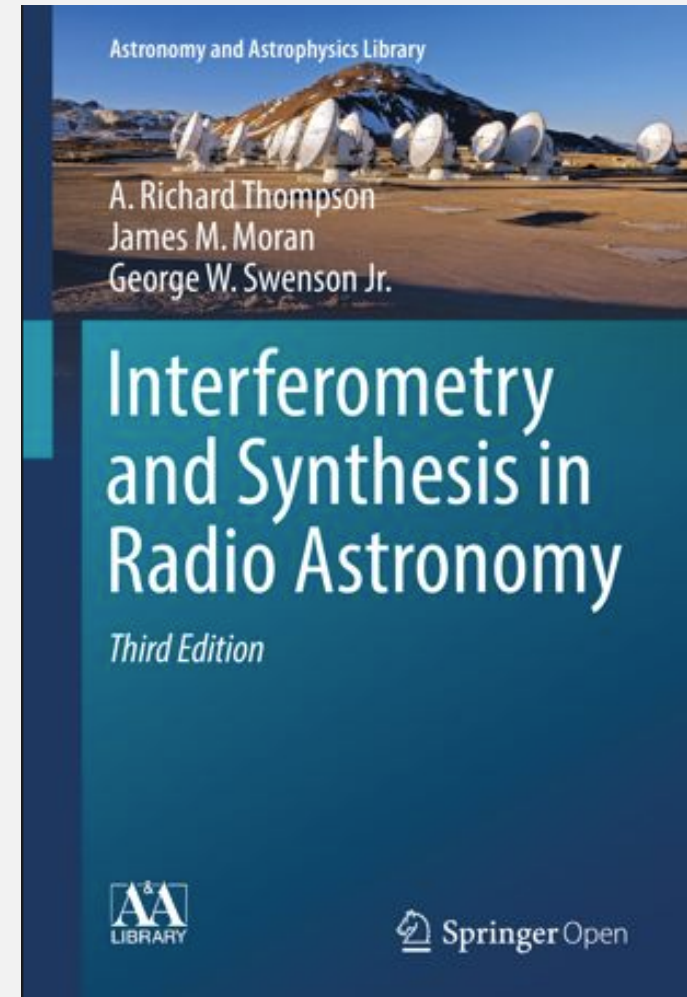
Judaism: Tanakh

Radio Interferometry: Thompson, Moran & Swenson

**Third Edition available for FREE!**

<https://www.springer.com/gp/book/978331944291> In General

<https://link.springer.com/book/10.1007%2F978-3-319-44431-4> In Mexico



# At least three different ways to view/analyze interferometers...

1. An antenna with non-contiguous parts  
or a single huge antenna, with much of the surface missing.  
Analysis in terms of 'fringes'; appropriate for adding interferometers
2. Electric field of the astronomical source can be measured in a plane on the sky, by locating two antennas in a parallel plane on the Earth.  
The antennas measure the correlation function of the field, which is the Fourier Transform of the source brightness distribution.
3. Two antennas receiving a signal from a point source from direction  $\psi$ , with an excess travel distance  $B \cos \psi$  to the further antenna. This produces an interference pattern with a 'fringe phase' which can be 'stopped' by inserting a delay into the signal path.

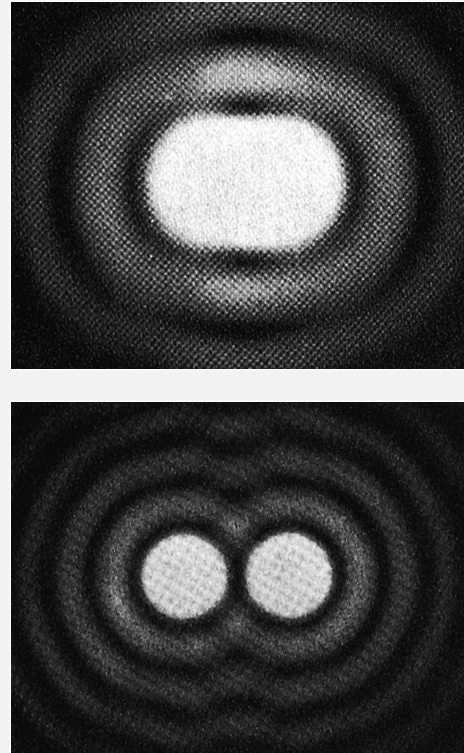
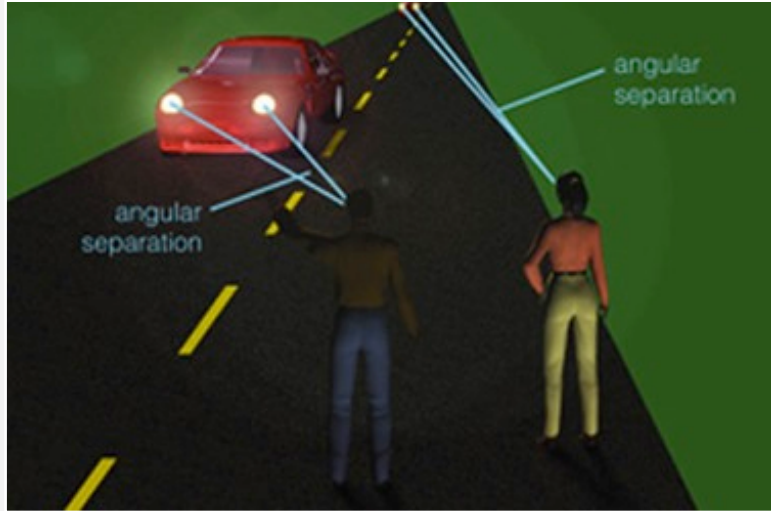
# Why use interferometers at all?

# The Problem of Angular Resolution

$$\Theta \sim \lambda/D$$

Angle ~ wavelength / telescope diameter

For single-dish telescopes, this is both the Field-of-View and the Angular Resolution



## Optical Telescopes

$$\frac{500 \text{ nm}}{8 \text{ m}} = 0.00000006$$

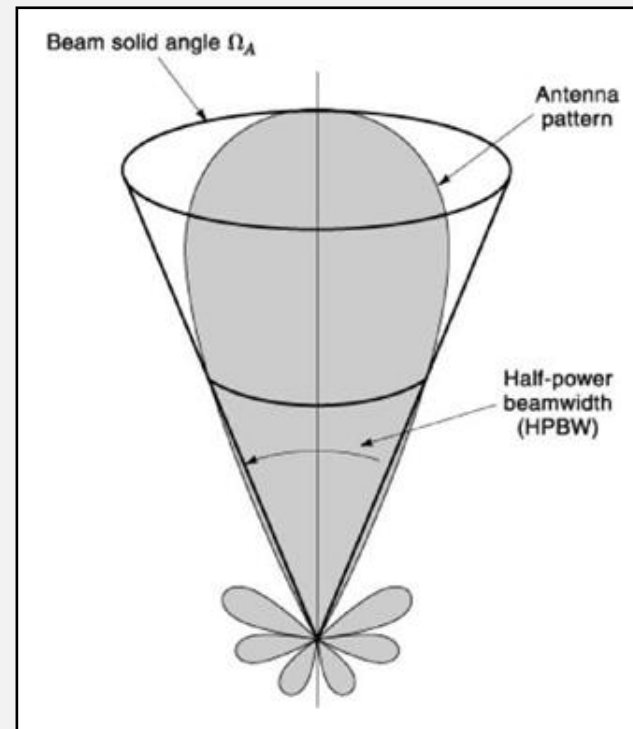
Earth-based limit about one arcsec  
(seeing limited, not diffraction limited)

## Radio Telescopes

$$\frac{1 \text{ cm}}{30 \text{ m}} = 0.0003$$

About one arcmin

About 70,000 AU at 1 kpc  
or about 1000X the Solar System

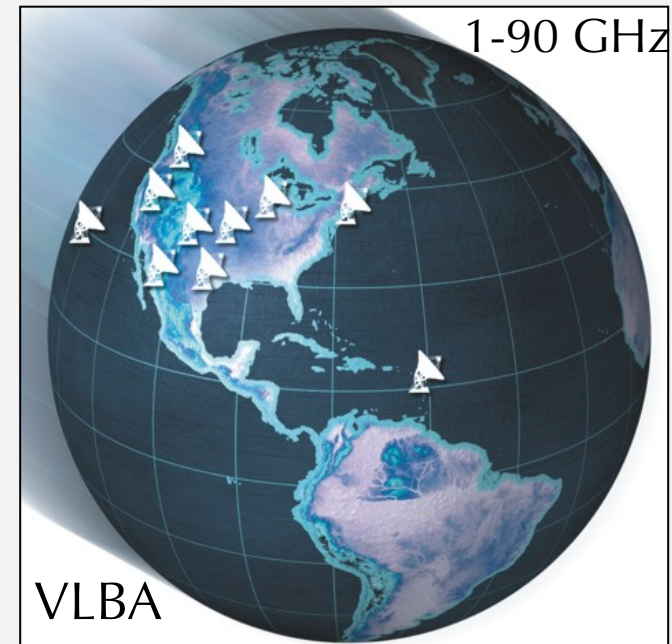




$$\Theta \sim \lambda/B$$

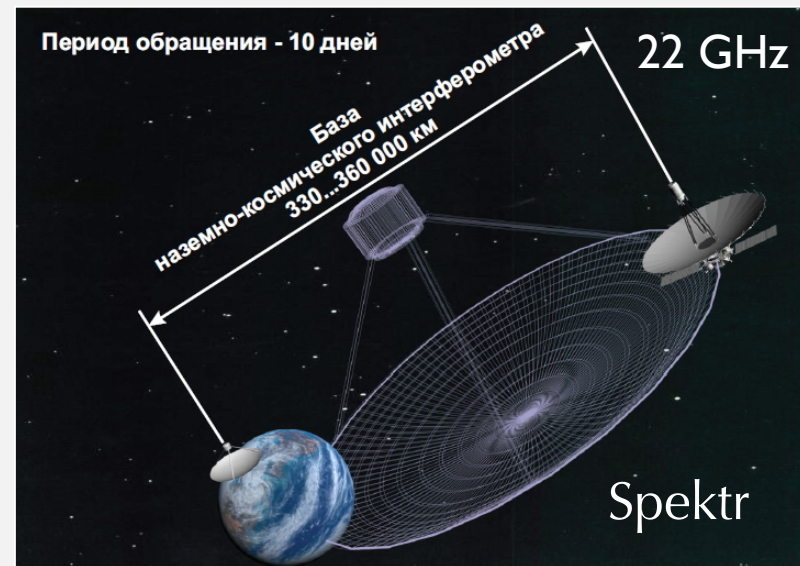
B is the *baseline*, or the *separation between antennas*

Micro-arcsec resolution is possible

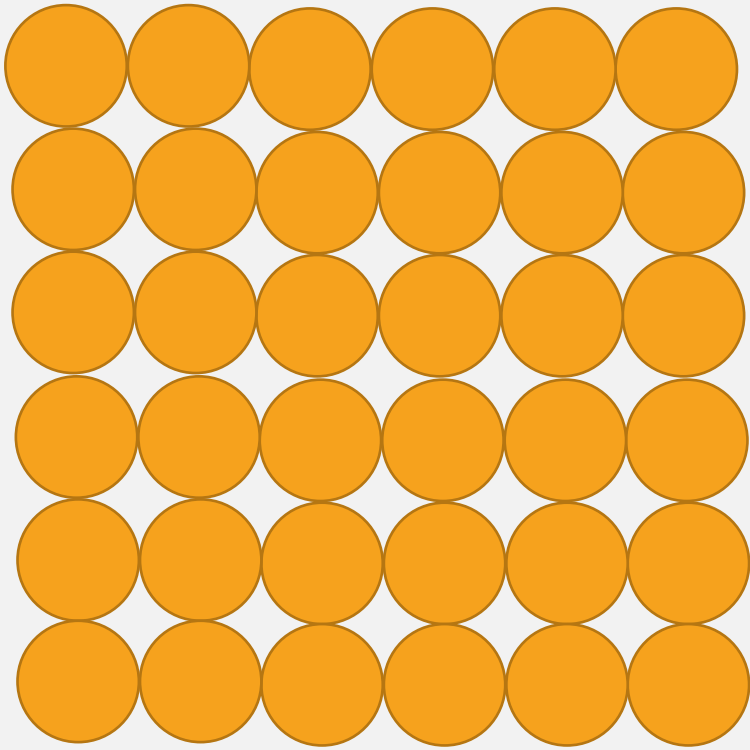


Two beam sizes now important:

- 1) Primary beam (field of view) from diameter D
- 2) Synthesized beam (resolution) from spacing B

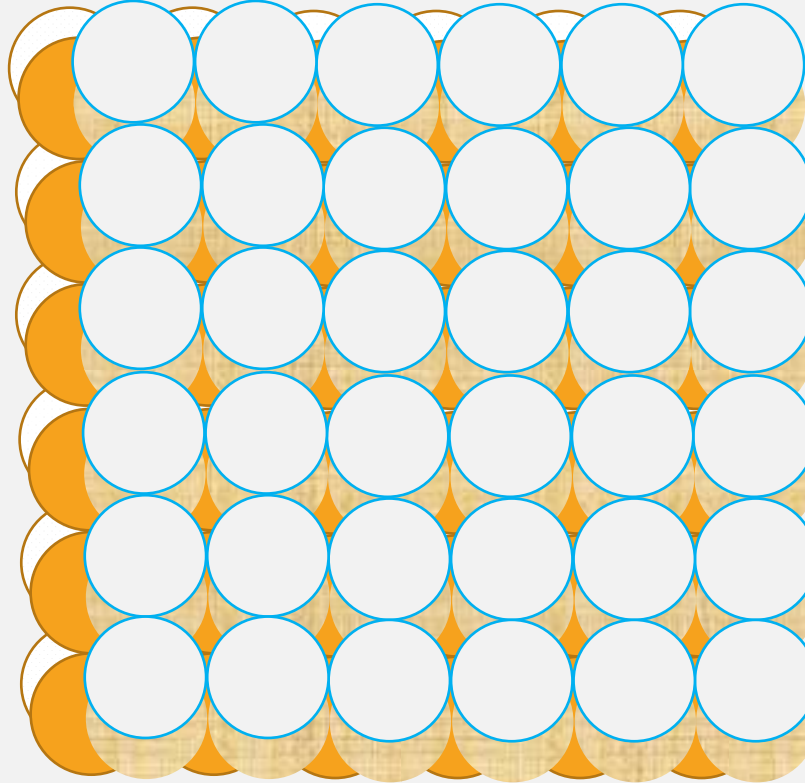


# Making Images with Single-Dish Telescopes



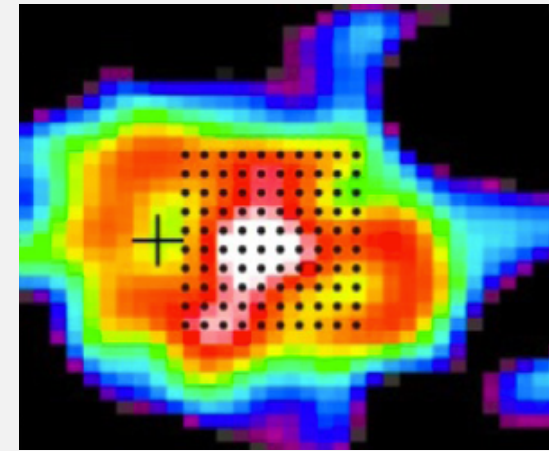
6 x 6 pixels from 36 pointings  
vs 2048 x 2048 in optical CCDs!

FPA: Focal Plane Arrays (multi-pixel feeds)  
Speed both multiple pointings and OTF mapping



Even worse!  
 $4 \times 36 = 144$  pointings  
for 'half-beam' sampling  
that satisfies the  
Nyquist Criterion

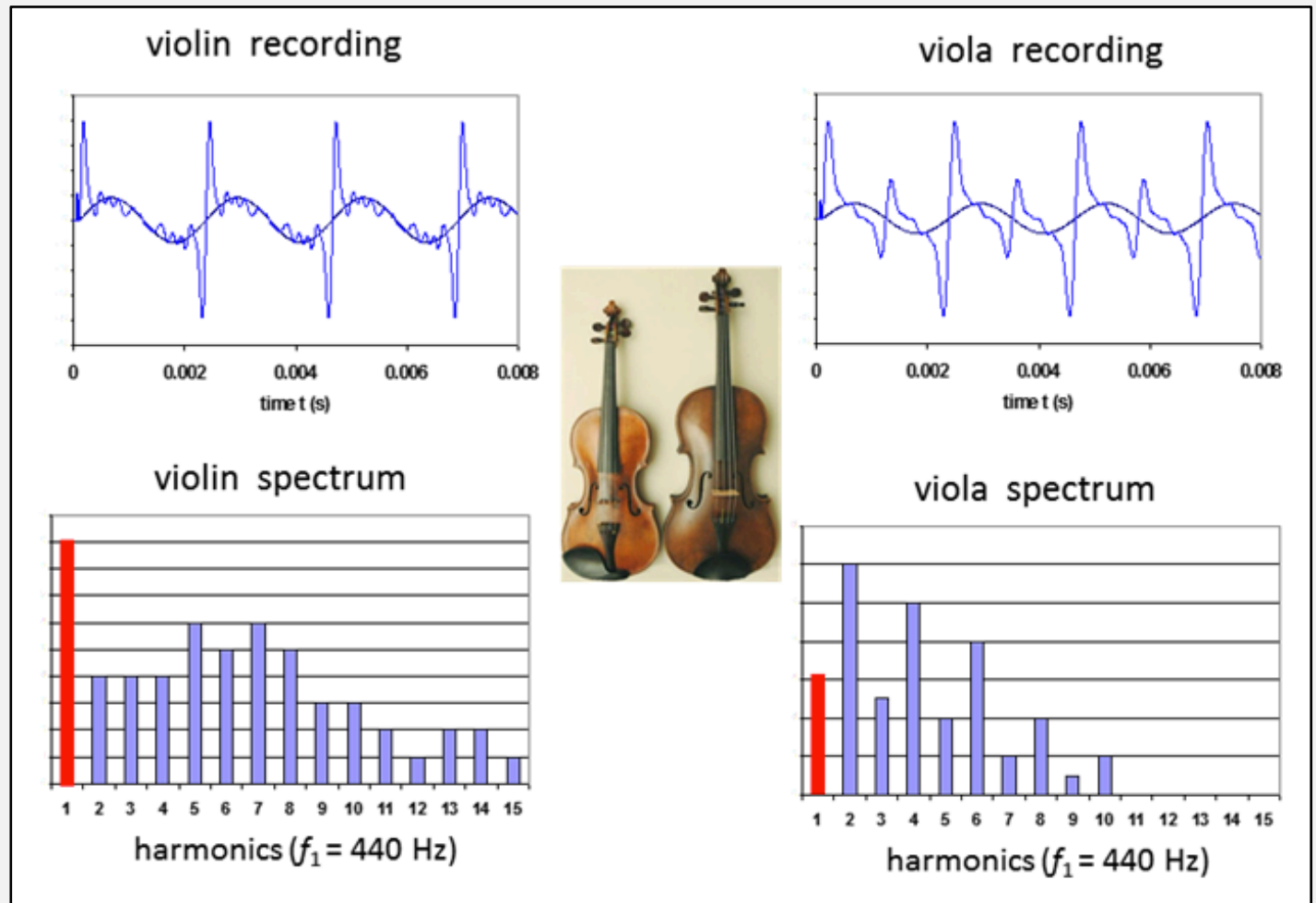
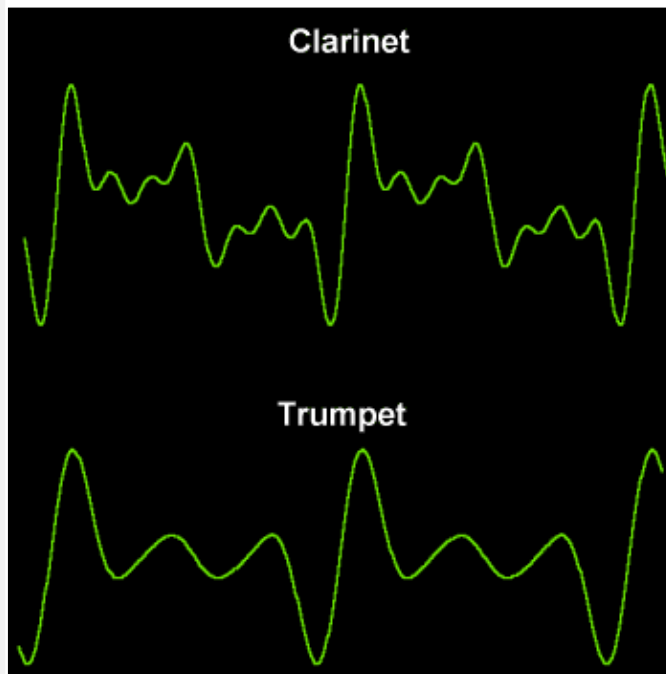
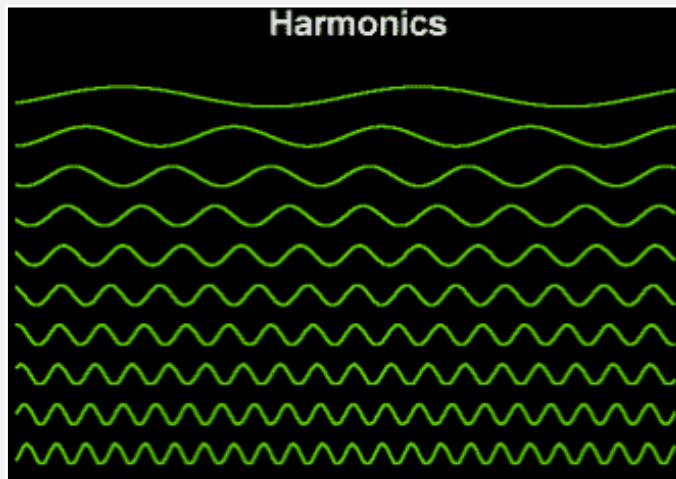
OTF: On The Fly mapping  
via raster scanning is  
another option



But how do we  
make images with  
an interferometer?

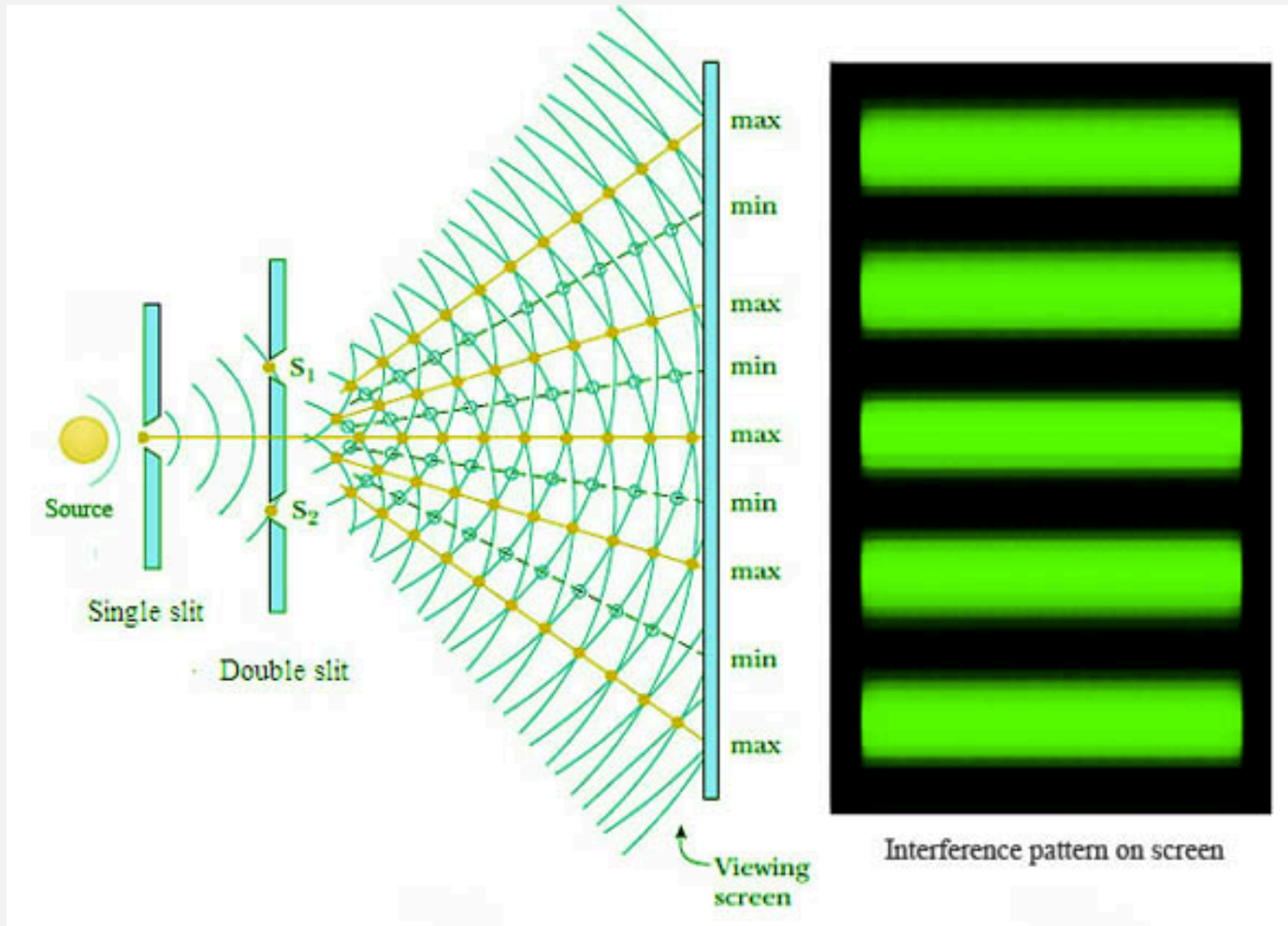
Problem: How to form something (anything!) from sines and cosines?

Solution is well-known from music: Fourier Synthesis



For images we'll use *spatial*, rather than *temporal* frequencies

Antenna pairs act so as to form a two-slit interference pattern



Maxima occur when

$$d \sin \theta = n \lambda$$

Spacing of maxima is:  $y = n \frac{\lambda D}{d}$

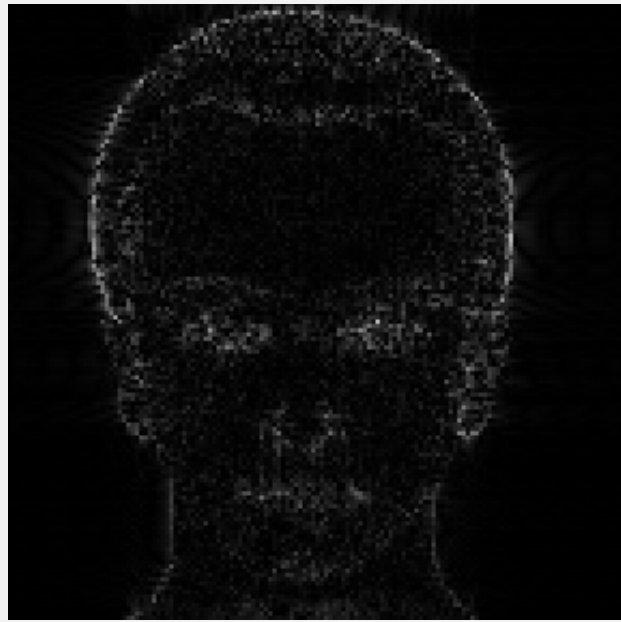
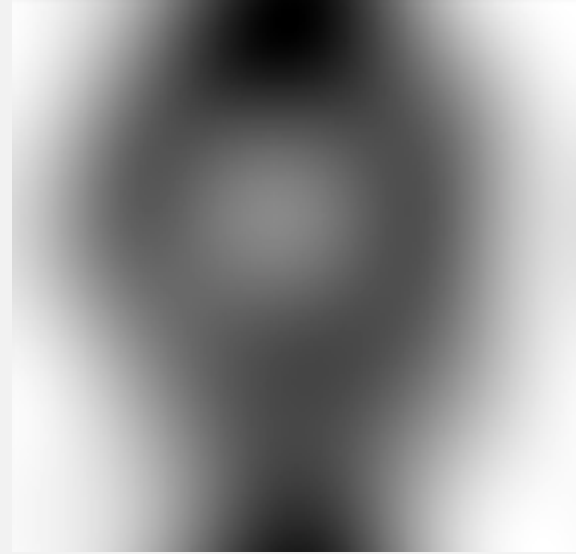
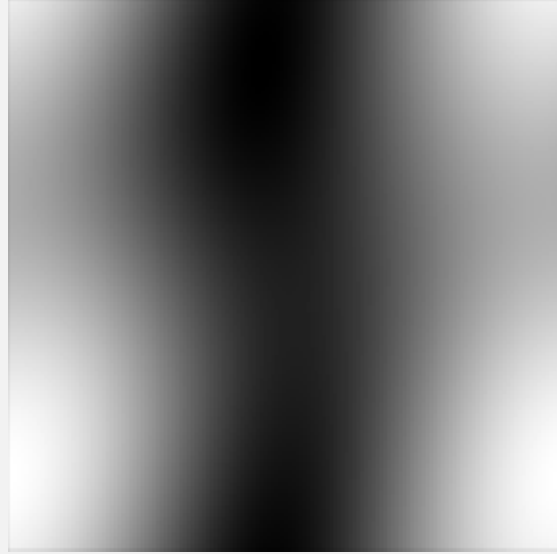
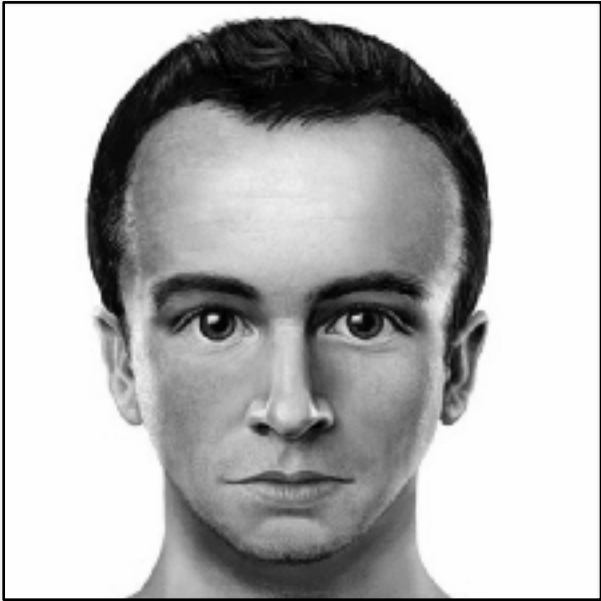
Small slit spacing gives a large pattern spacing

Large slit spacing gives a small pattern spacing

Closely spaced antennas sample low spatial frequencies: they see big things  
Widely spaced antennas sample high spatial frequencies: they see small things



## Low Spatial Frequencies Show Larger Shapes



High Spatial Frequencies  
Show Fine Details but lose  
The Bigger Shapes

# Getting the Fourier Components

$$I(x, y) = \iint V(u, v) e^{-2\pi i(ux+vy)} du dv$$

Sky brightness = Fourier Transform of the Visibilities in  $uv$  space

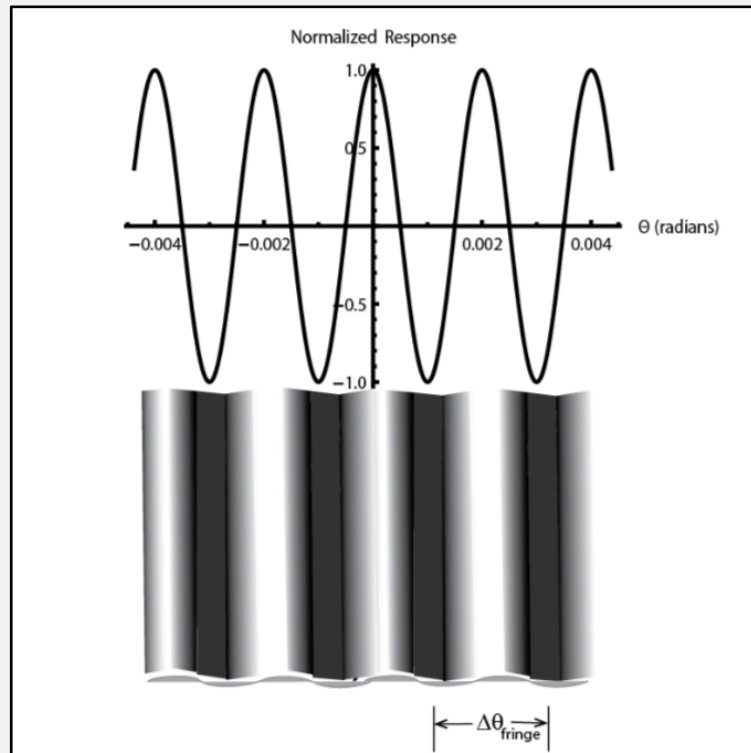
$(x, y)$  in radians       $u = \frac{b_x}{\lambda}$        $v = \frac{b_y}{\lambda}$        $(u, v)$  in kilo or mega wavelengths

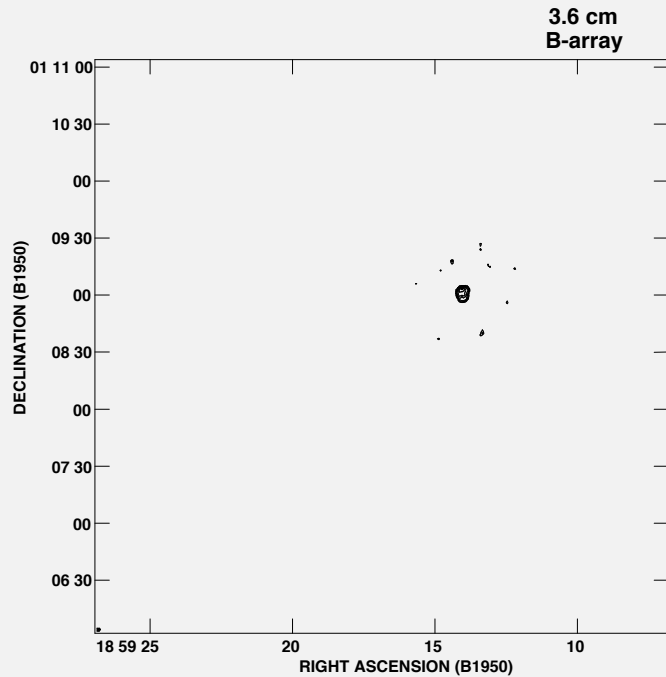
Visibilities are the cross-correlation of the antenna signals, corrected for the fringe function

$$\langle E_1 \cdot E_2 \rangle = \frac{E_0^2}{2} \cos \left( 2\pi \frac{b}{\lambda} \sin \omega_E t \right)$$

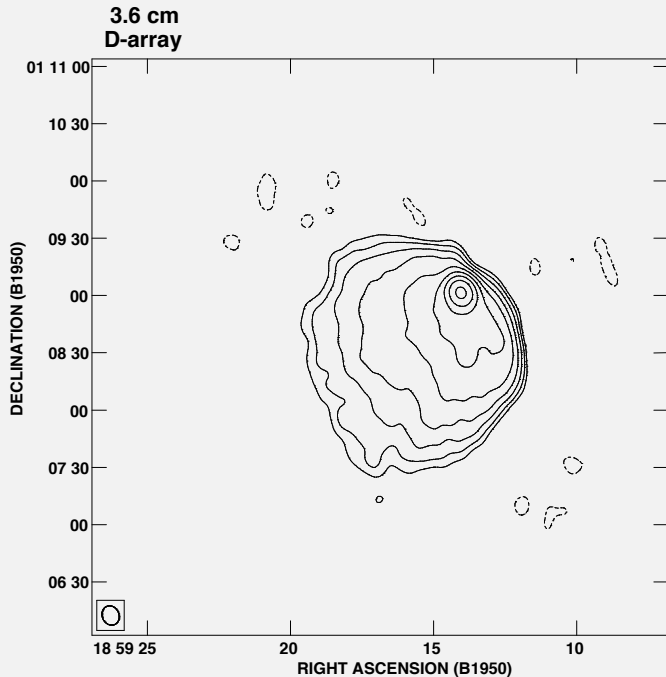
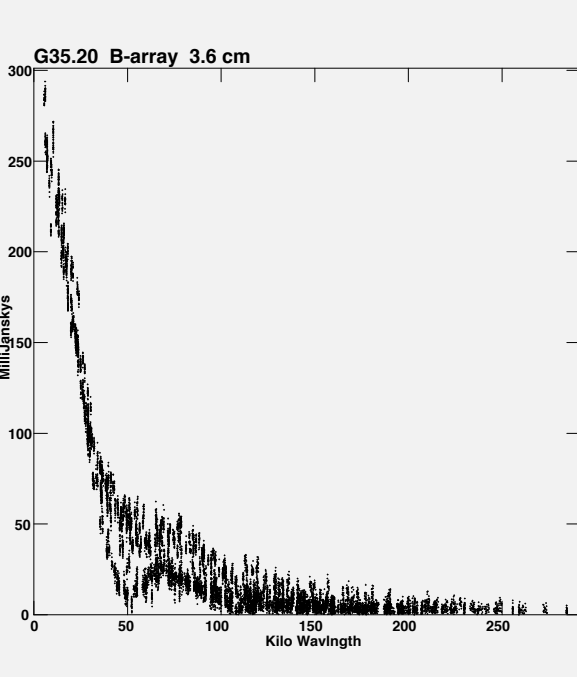
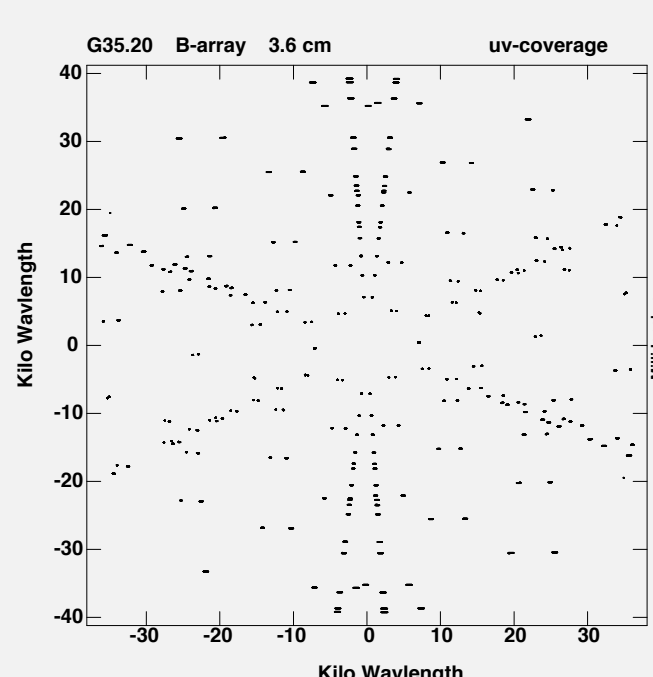
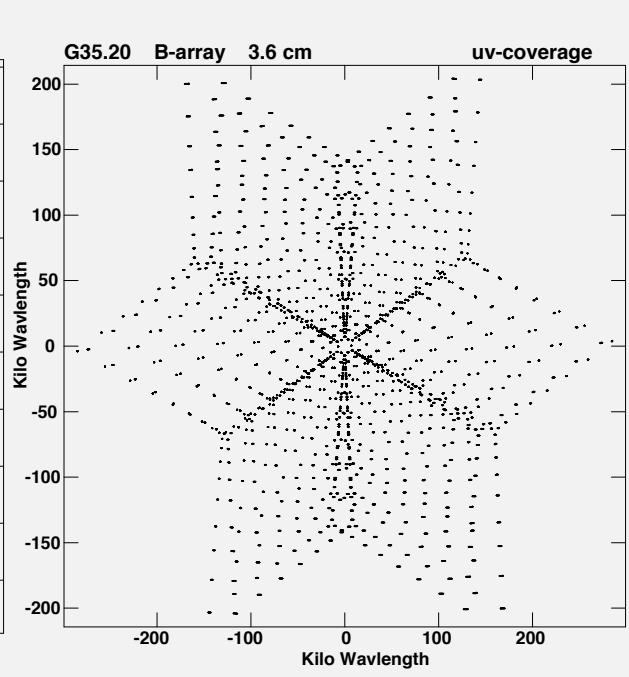


The fringe function,  
due to Earth Rotation

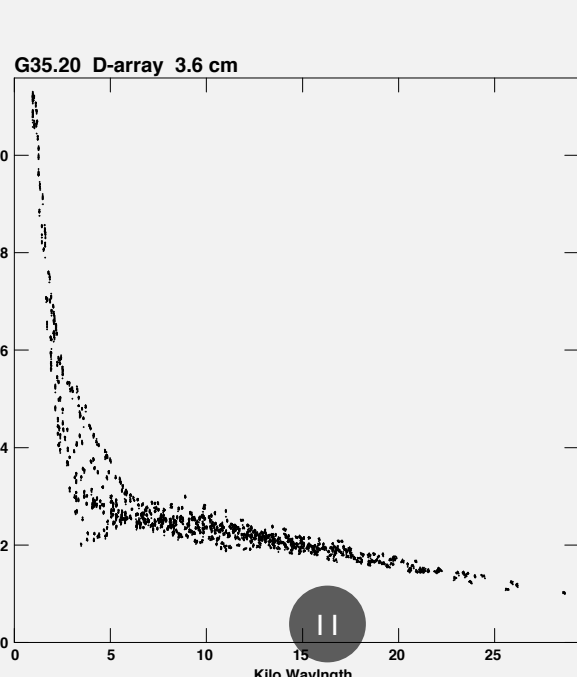
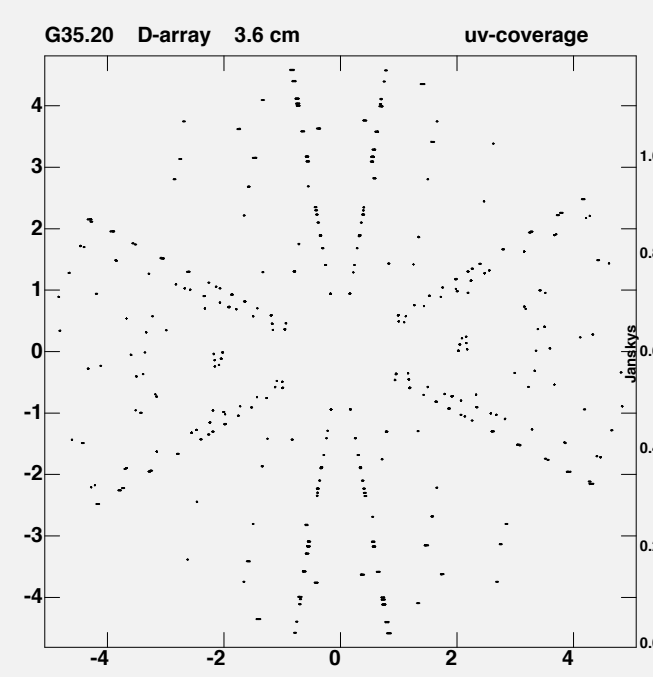
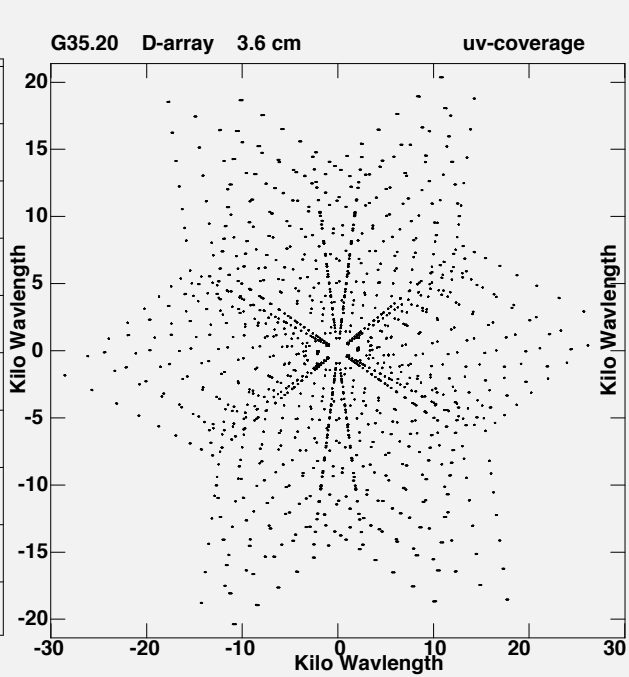




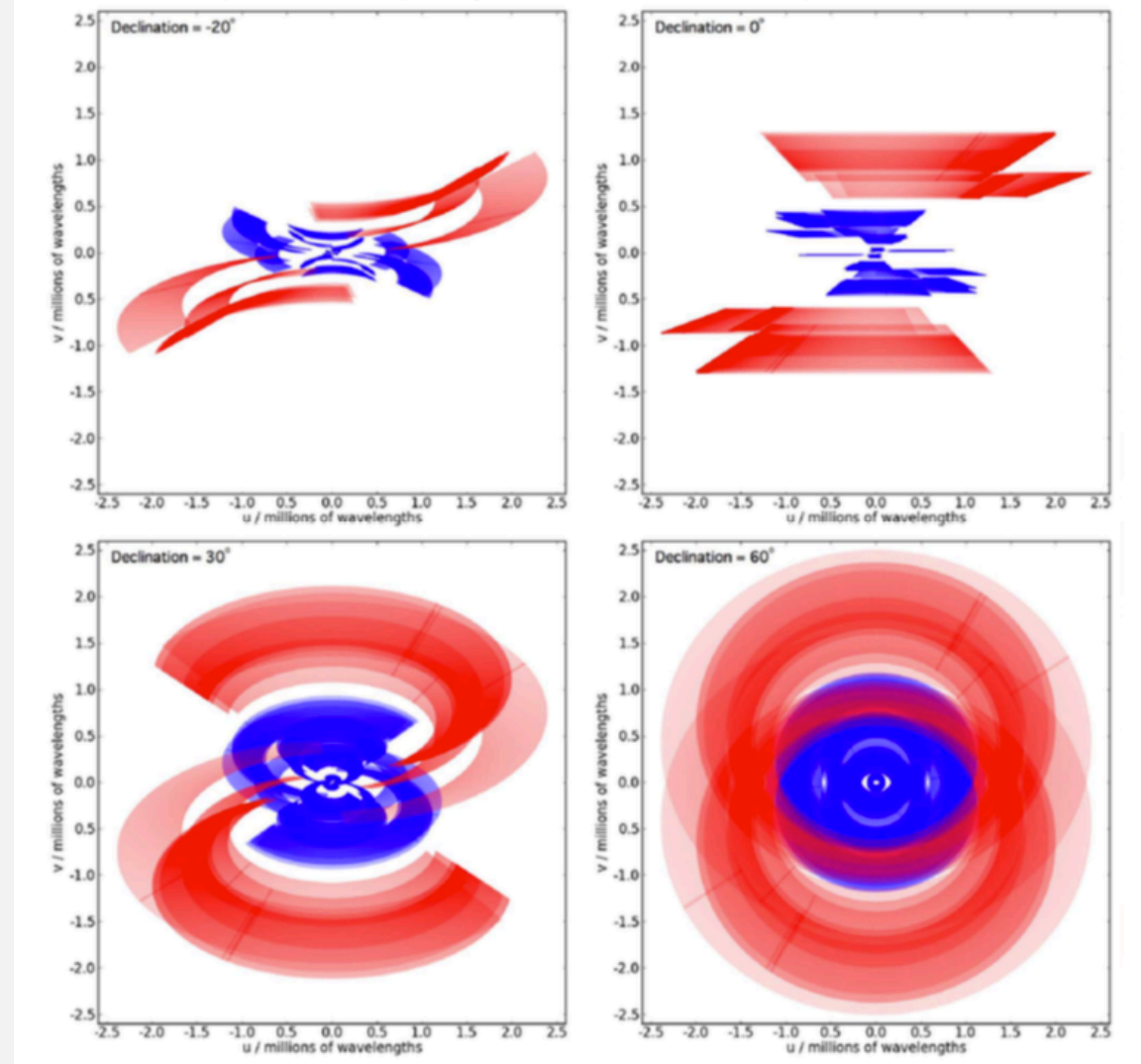
Peak continuum flux density = 0.171 Jy/beam  
 Levels = 6.60E-04 \* (-4, 4, 8, 16, 32, 64, 128, 256)



Peak continuum flux density = 2.480 Jy/beam  
 Levels = 2.0E-03 \* (-4, 4, 8, 16, 32, 64, 128, 256, 512, 1024)



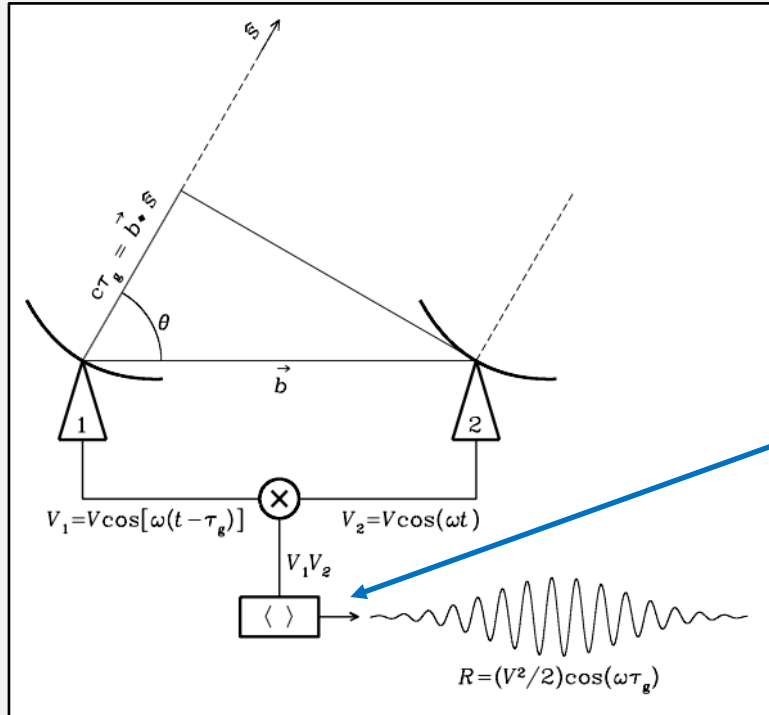
# The effects on uv coverage by adding the Goonhilly antenna to e-MERLIN



Heywood et al. 2011

Better coverage of the  $uv$  plane gives better images!

If we have *Radio Interferometry*, Why do we need VLBI? and What's so special about it?



*Connected-element* interferometers have a *real-time* connection between the antennas

The correlation of the antenna signals occurs in real-time

But what if we need baselines so long that real-time connections don't work?

Suppose we need 1 milli-arcsec resolution at 30 GHz

Solution is to *record the data* at each antenna, for later correlation

$$\theta \sim \frac{\lambda}{B}$$

1 mas is  $4.85 \times 10^{-9}$  radians  
30 GHz is 1 cm wavelength

$$4.85 \times 10^{-9} = \frac{1 \text{ cm}}{2000 \text{ km}}$$

# VLBI vs Connected-Element Interferometry: What's so different?

- A) Each antenna needs an atomic clock
- B) High brightness temperature sources are needed
- C) Only a small field of view can be imaged (BW smearing is extreme)

## A) Greater Hardware Requirements

The correlation process requires very precise time-stamping of the data, hence the need for atomic clocks at each station

Lots of disk storage capacity is needed

Shipping & Correlation  
Infrastructure is needed

Year	Sustainable Rate (Mbps)	Disk purchase (TB)	Disk cost (\$/GB)	Other costs (\$/GB)	~ cost
1	256	825	1	.15	\$1.0M
2	512	825	.75	.15	\$0.8M
3	1024	1750	.50	.10	\$1.0M
Total		3300			\$2.8M

## B) What's the deal with brightness temperature?

Resolution depends on antenna *separation*

Sensitivity depends on antenna *area*

VLA in D-array

$$27 \times 25 \frac{\pi^2}{4} = 1664 \text{ m}^2$$

$$\pi (0.5 \text{ km})^2 = 785,000 \text{ m}^2$$

$$10^{-3}$$

VLBA

$$10 \times 25 \frac{\pi^2}{4} = 620 \text{ m}^2$$

$$\pi (4000 \text{ km})^2 = 5 \times 10^{13} \text{ m}^2$$

$$10^{-11}$$

True collecting area

Spanning area

Fractional area



High source brightness temperature is needed  $\sim 10^6 \text{ K}$

Not all astronomical sources can be detected!

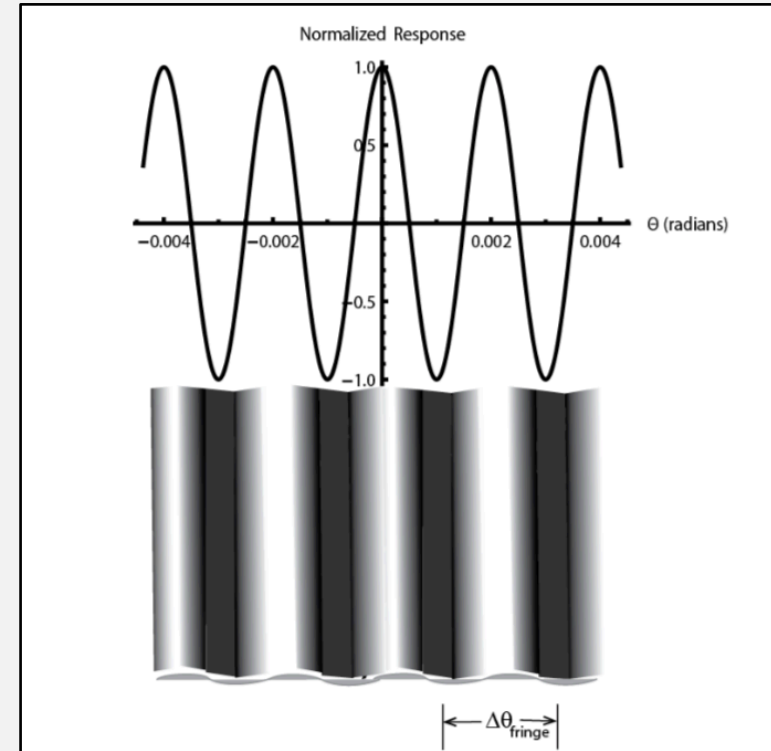
## C) Why the small field of view?

Bandwidth and Time Smearing  
limit the angular area over which  
an image can be made

Typical map dimensions are  $< 1$  arcsec

Multiple positions may be mapped with multiple correlations

Software correlators allow these positions to be  
mapped *simultaneously*

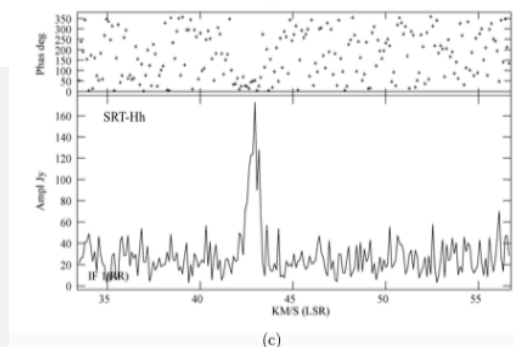
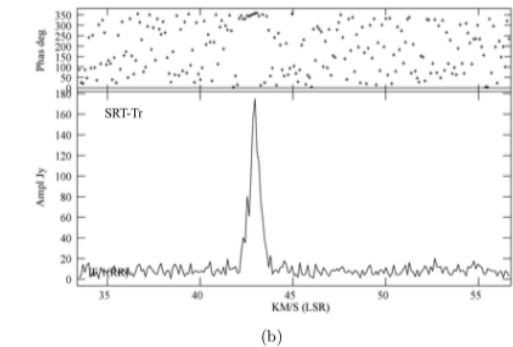
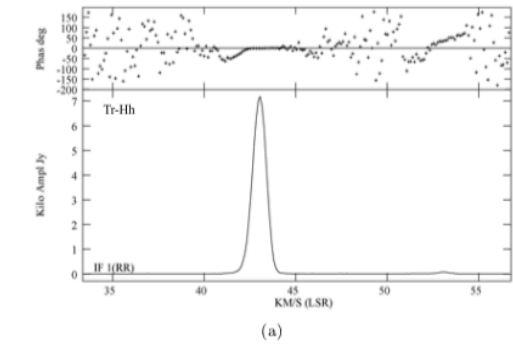
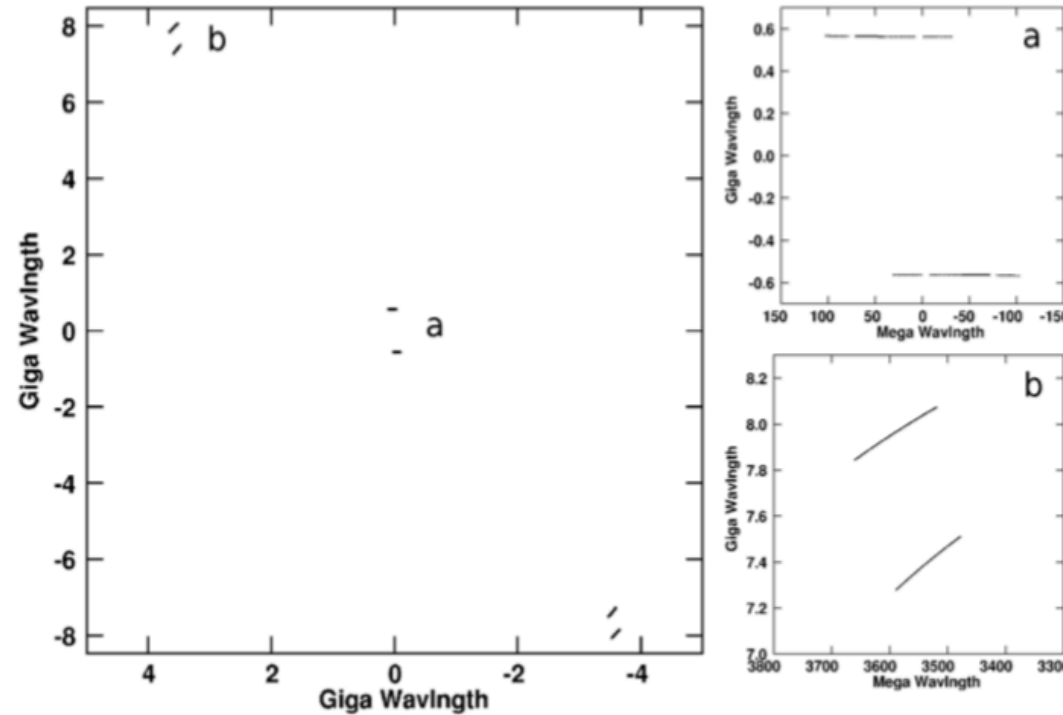




# How many antennas do you need to make an image?

1. Radio Astron (in Space)
2. Torun (in Poland)
3. Hartebeesthoek (in South Africa)

With such poor uv coverage imaging is not really possible; the visibilities must be modeled



Bayandina et al. (2019, accepted)

No hard rule, but  $\geq 6$  or 8 needed for imaging

3 antennas needed for phase closure

4 antennas needed for amplitude closure

